Buyer-Supplier Interaction, Asset Specificity, and Product Choice[1]

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Abstract

Firms often differentiate their products through the use of inputs that are differentiated or specialized. The goal of this paper is to explore how the demand for specific investments may affect product variety in a bilateral duopolistic industry. I develop a model where the degree of specificity of investments is endogenously determined through the product choices of both downstream buyers and upstream suppliers of inputs. The existence of non-contractible investments results in the emergence of a single supplier of a non-specialized input. However, as the importance of having specific inputs increases, fully specialized suppliers may emerge. The buyers choose to increase their own competition by producing more similar products only in cases when the inputs are non-specialized. They do so to increase the investment incentives of the suppliers.

JEL Classification: L11, L23, R12

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1 Introduction

Product choice is one of the most fundamental decisions firms have to make. Standard economic reasoning suggests that firms would want to produce differentiated products in order to reduce the competition between them.[1] This reasoning rests on the assumption that once a firm makes its product choice, it can easily find the inputs it needs to produce that product variety. In reality, it is often the case that the production of different varieties requires different inputs and firms are often restricted in their product choice by the availability of inputs.[2] Hence, the product choices of downstream firms cannot be analyzed in isolation from the product choices of upstream firms. Suppliers may have to make product-specific investments to develop the required inputs. In an environment of incomplete contracts, depending on the specificity of the suppliers' investments, this suggests that there may be a potential hold-up problem.

The goal of this paper is to explore how the demand for specific investments may affect product variety in a bilateral duopolistic industry. I consider a model of differentiated products where specific investments arise because there is an ideal input type corresponding to each possible variety of a final good. The downstream firms compete against each other in the familiar Hotelling model. They can reduce their fixed costs of production by procuring inputs from the suppliers in the upstream market. The suppliers choose what type of input to produce and how much to invest in its development after the downstream firms choose which final goods to produce. Due to contractual incompleteness, the suppliers face a potential hold-up problem. The input has no value if the investment is zero. I assume that if the downstream buyer uses an input type that is different from its ideal input type, the value created is decreasing in the distance between the product choices of the buyer and the supplier. After the suppliers make their decisions, the input prices are determined by bilateral negotiations, which is a common occurrence in markets for intermediate goods. In the bargaining process, the buyers' payoff depends on whether they have access to alternative suppliers they can trade with and the suppliers' payoff depends on whether they have alternative uses for their inputs.

[1] Since the seminal work of Hotelling (1929), a lot of attention has been given to the question of product choice. See Thisse and Norman (1995) for a survey of the literature.

[2] It is emphasized in the product management literature that the distinctiveness of two final goods may significantly depend on the distinctiveness of the inputs used in their production (Robertson and Ulrich, 1998). Krishnan and Ulrich (2001) explain the close links that exist between firms' choices of inputs and their decisions regarding which product variety to produce.

In this context, the analysis focuses on two questions. First, when do suppliers choose to produce specialized inputs? Second, when do buyers choose to be less than maximally differentiated? The model allows us to analyze the feedback mechanism between the buyers' and the suppliers' decisions. The degree of product differentiation is endogenously determined in both the downstream and upstream markets. While making their product choices, the downstream firms take into account the impact of their decisions on the upstream firms. Since the suppliers make their decisions after the buyers, their decisions reflect how specialized they would like to be. Both the buyers' and the suppliers' product choices jointly determine the degree of asset specificity in the market. The degree of specificity of an input to a given buyer in the downstream market depends on how suitable the input is for the purposes of that buyer as well as how suitable it is for the purposes of alternative buyers.

The analysis reveals several interesting findings. I start by considering the case where the buyers are flexible in using inputs that are not perfectly suited for their purposes (i.e., the value loss from not having fully specialized inputs is relatively low). In this case, only one supplier serves the downstream market in equilibrium. With only a single supplier in the market, the buyers do not have access to specialized inputs, but the supplier invests more in the development of the input. Thus, imperfect contracting creates a trade-off between the supply of fully specialized inputs and the supply of highly developed inputs. The buyers may choose to be less than maximally differentiated because when served by a single supplier, they have incentives to move closer to that supplier to make its inputs more suitable for their purposes and to increase the supplier's investment incentives. Hence, in contrast with the maximum differentiation result of d'Aspremont et al. (1979), firms may not always choose to be maximally differentiated in an effort to decrease the competition between themselves.

These results suggest one explanation for the use of common inputs, i.e., platform sharing, which has become a common practice in the automobile industry (Robertson and Ulrich, 1998). According to my model, firms may choose to produce similar products and, thus, use similar inputs in order to improve their suppliers' investment incentives.

I next consider how an increase in the disutility that buyers get from using an input type that is different from their ideal input type affects the results. The analysis reveals that as the importance of having specific inputs increases, fully specialized suppliers may emerge. This is because as it becomes more costly for the buyers to use an input variety that is different from their ideal variety, they prefer to locate such that there will be two suppliers in the market. This always results in maximum differentiation.

Thus, the flexibility of buyers in using the inputs of different suppliers may affect the structure of vertically-integrated industries. Bonaccorsi and Giuri (2001) explain how the ability to benefit from economies of scale and scope have resulted in distinct network structures in the turbojet and turboprop industries. They show that in the turboprop industry, the network structure is more fragmented because of a lower ability to adapt different engine programs to different aircraft programs.[3]

This paper contributes to both the literature on product differentiation and the literature on the hold-up problem by bringing them together in a single model and showing the interdependence between them. Although there is an extensive literature in industrial organization on product differentiation, the focus has almost entirely been on price and product competition in the downstream market.[4] Little attention has been paid to the links between the downstream and upstream markets. Three exceptions are Pepall and Norman (2001), Belleflamme and Toulemonde (2003), and Ghosh and Morita (2006). Pepall and Norman (2001) consider a model where downstream firms can produce differentiated products by using different combinations of the differentiated inputs produced by the upstream suppliers. Their goal is to study the profitability of different vertical alliances. Belleflamme and Toulemonde (2003) illustrate that downstream and upstream firms may choose to agglomerate in an effort to decrease their costs because the upstream firms experience economies of scale. Ghosh and Morita (2006) show that manufacturers may choose to produce similar products to have access to a greater pool of suppliers and reduce their procurement costs. None of these papers considers the investment choices of the suppliers. Hence, this paper differs by considering the impact of specific investments on the product choices of both upstream and downstream firms in the context of incomplete contracts.

In the literature on the hold-up problem, my analysis of the endogenous choice of input specificity is related to Choi and Yi (2000), Church and Gandal (2000), McLaren (2000), and Grossman and Helpman (2002). A core element of these papers is that an integrated supplier chooses to produce a more specific input than an unintegrated supplier.[5] Thus, whereas these

[3] Eaton and Schmitt (1994) also analyze the impact of flexible manufacturing on market structure. However, they consider the product choices of downstream firms only.

[4] Current theories of horizontal product differentiation have been very much influenced by the "Main Street" model of Hotelling (1929). Ever since d'Aspremont et al. (1979) challenged the "Principle of Minimum Differentiation" established by Hotelling (1929), several papers have argued for minimum differentiation based on assumptions such as collusion or consumer heterogeneity. See, for example, de Palma et al. (1985) and Friedman and Thisse (1993). The incentives to agglomerate have also been analyzed in the economic geography literature. See Fujita and Thisse (1996) for a discussion.

[5] Riordan and Williamson (1985) also analyze the impact of asset specificity on firm boundaries. However,

papers study how vertical integration may affect the choice of input specificity, I study how the links between downstream and upstream firms may affect the choice of input specificity. The downstream firms' locations are taken as given in these four papers. In contrast, I emphasize how the downstream firms may strategically commit to competition in order to alleviate lock-in on the part of their suppliers. In this respect, the paper is also related to the literature where firms may undertake strategies in order to commit themselves to leave their buyers or suppliers with higher rents in the future. For example, Williamson (1983) explains how buyers may use hostages to support exchanges and encourage investment in specific technologies. Farrell and Gallini (1988) and Shepard (1987) analyze the strategic use of commitment to competition in order to alleviate consumer lock-in.[6]

The paper proceeds as follows. The details of the model are presented in the next section. Section 3 considers the case where the buyers are flexible in using alternative inputs. Section 4 extends the analysis in Section 3 by considering the impact of an increase in the importance of having specialized inputs. Section 5 concludes by suggesting avenues for future research. All proofs are in the Appendix.

2 Model

To capture the impact of specific investments on product variety in a bilateral duopoly, I extend the standard Hotelling model of spatial competition by incorporating upstream suppliers of inputs. Two downstream firms compete in a differentiated final goods market and two upstream firms sell inputs to the downstream firms. The varieties of the final good are differentiated in two respects. First, consumers regard different products as imperfect substitutes. Second, each type of final good has an ideal input type associated with it. I refer to the downstream firms as buyers (of inputs) and the upstream firms as suppliers.

The timing of the game is as follows. In the first stage, the two buyers, B_1 and B_2, choose the varieties of the final good they would like to produce. Production entails a fixed cost of production. Each buyer can use a single unit of an input developed by a supplier to decrease its fixed cost of production. In the second stage, the two upstream suppliers, S_1 and S_2, choose both which input type to produce and how much to invest in its development. Due

they do not consider how the decisions of other firms in the industry may affect the behavior of a specific firm.

[6] See also Inderst and Wey (2003) who study horizontal mergers in bilaterally oligopolistic industries. They show that downstream firms may merge to affect their suppliers' incentives to make non-contractible investments.

to contractual incompleteness, the terms of trade are determined ex post through bilateral negotiations in the third stage. Finally, in the fourth stage, the buyers compete in the final goods market by choosing prices simultaneously and make sales.

To capture the consumers' demand for differentiated products, I employ the setting of d'Aspremont et al. (1979), which modifies the standard Hotelling (1929) model of spatial competition by assuming quadratic consumer transportation costs. The market region is described by the unit interval $[0, 1]$. There is a unit mass of consumers who are uniformly distributed along this interval. Each consumer's location, denoted by $x \in [0, 1]$, represents that consumer's ideal variety. The two downstream buyers, B_1 and B_2, locate at distances b_1 and b_2 from the two ends of the unit line. Without loss of generality, I assume that B_1 is located to the left of B_2: $0 \leq b_1 \leq 1 - b_2 \leq 1$.

Each consumer purchases exactly one unit from the firm offering it at the lower effective price, namely the mill price plus the transportation cost. This implies that a consumer of type x purchasing from B_1 and B_2 gets utility $U(x, b_1, p_1) = S - p_1 - t(x - b_1)^2$ and $U(x, b_2, p_2) = S - p_2 - t(x - (1 - b_2))^2$ respectively, where S represents the gross surplus from consumption, p_1 and p_2 stand for the prices at which the buyers offer their goods, and t is a measure of consumer loyalty. I assume S is sufficiently high so that all consumers buy. Disutility costs vary quadratically with the distance between the product produced by the firm and that which is most preferred by the consumer. As shown in d'Aspremont et al. (1979), this assumption prevents discontinuities in the profit functions of the firms.

Production of the final goods entails a fixed cost and entry into the market implies a commitment to incur the fixed cost of production. This assumption implies that the buyers make the production decision at the same time as the location decision since they cannot avoid the fixed cost by not producing later in the game. To illustrate, the fixed cost of production may be the cost of abiding by some government regulation. For example, in the mining industry, firms in most jurisdictions are subject to clean-up costs, which are the costs of securing, decommissioning, and dismantling of mining facilities. Firms commit to incur these costs when they open a mining facility in a specific location.[7] However, they can obtain specialized technologies or machineries which may reduce the contamination during the production process or the costs of cleaning up after the production process.[8]

[7] I am grateful to Daniel Piccinin for suggesting this example. The mine clean-up procedure can involve substantial costs and can take years to complete. See, for example, the Columbus Dispatch (2006).

[8] Alternatively, one can interpret the current set-up as a situation where the buyers are committed to production due to contracts they have signed prior to entering the market. For example, the buyers may

Before entry, the profit functions of the buyers have the general form

$$\pi_i(p_1, p_2) = (p_i - c) D_i(p_1, p_2) - F_i, \quad i = 1, 2 \tag{1}$$

where $D_i(p_1, p_2)$ stands for the demand for buyer B_i's product, and c and F_i denote the marginal and fixed cost of production respectively. As mentioned above, the buyers can reduce the fixed cost of production with the purchase of a single unit of a customized, location-specific input.[9] If B_i does not purchase any inputs, $F_i = K$. Thus, I assume that the firms are initially symmetric in their costs of production. More generally, the specific value F_i takes is revealed after the input purchases and prices are determined by bilateral negotiations in the input market as described below. In the following analysis, I assume F_i is sufficiently low such that both buyers find it profitable to enter the market.

To obtain the inputs they need, the buyers contact the suppliers in the upstream market after making their location decisions. The location of the final good in the market interval represents the specification of the input needed in its production. Hence, the unit interval represents both the space of possible input types and the space of possible final goods. I assume that B_i contacts S_i, where $i = 1, 2$, and provides the supplier with specific information on the characteristics of the input it requires. The information provided by the buyers allows the suppliers to locate anywhere they choose in the input space.

The benefit each buyer receives from the input it uses depends on how close the input is to the buyer's ideal variety as well as how much effort the supplier exerts in its development. I assume that due to the uncertain nature of the innovation process, the parties cannot sign enforceable ex ante contracts for the delivery of a specific input (Aghion and Tirole, 1994). The precise nature of the required input is revealed only ex post and the parties cannot commit to refrain from renegotiation.[10] As is well known, the absence of ex ante contracts results in the hold-up problem in the investment stage. Since my goal is to analyze the impact of the hold-up problem on the firms' product choices, I assume that vertical integration is not a viable way of dealing with the potential underinvestment problem.

be construction firms which have signed contracts with two neighboring governments to build, operate, and transfer two potentially competing toll roads. In this case, the fixed cost of production may be the cost of building the road.

[9] Assuming that the inputs decrease the buyers' fixed cost of production, rather than possibly both their marginal and fixed costs of production, makes the analysis more tractable. I briefly discuss in Section 5 how changing this assumption would affect the analysis.

[10] For example, Helper and Levine (1992) state that in the automotive industry, contracts with suppliers are necessarily incomplete because of the complexity of the parts. They state that 'since engineering changes are common, the part actually produced by the supplier is often not the same as the part that was contracted for' (p. 567).

In the absence of ex ante contracts, the suppliers choose both the input variety they would like to produce and their effort level. They locate at distances s_1 and s_2 from the two ends of the unit line. A buyer located at b_i gets the maximum benefit from using the input produced by the supplier located at $s_i = b_i$. B_i can also use an input produced by a supplier located at a different point along the unit interval, but the benefit from using a less-than-ideal input decreases with the degree of differentiation between the locations of the buyer and the supplier. Specifically, since I would like to capture the interaction between the location and investment choices of the suppliers, I assume that the input B_i procures from S_i reduces its fixed cost of production by

$$e_i \left(1 - \gamma \left(b_i - s_i\right)^2\right), \tag{2}$$

where γ represents the importance of specificity in the input benefit function.[11] A lower value of γ corresponds to a higher flexibility in using input varieties that are different from the buyer's ideal variety.[12] This formulation assumes that the marginal benefit of effort depends on the distance between S_i and B_i, $|b_i - s_i|$. Hence, it implies that the suppliers' investment decisions are affected by their and the buyers' location choices. If $b_i = s_i$, B_i receives a fixed cost reduction of e_i. When $b_i \neq s_i$, the marginal disutility from using an input variety that is different from the ideal one is increasing in $|b_i - s_i|$. I assume the parameter values are such that the cost reduction that can be obtained from the inputs is less than or equal to K.

For the suppliers, the cost of exerting an effort level of e_i is $\frac{e_i^2}{2}$. In the input development stage, each supplier first develops a prototype. Once the suppliers develop the prototypes, they bring them to their potential customers and the partners negotiate the terms of trade. The cost of producing additional units of the input is zero.

For the determination of the input prices, I consider a bargaining game where the existence of alternative trading partners can potentially make a difference. The bargaining game consists of two stages.[13] In the first stage, B_i approaches the supplier it has contacted after choosing its location and they bargain over price. If the negotiations between B_i and S_i fail, they can try to find other partners in the second stage of bargaining and bargain with them.

[11] If B_i purchases the input from S_j instead, its fixed cost of production is reduced by $e_j \left(1 - \gamma \left(1 - s_j - b_i\right)^2\right)$.

[12] In the following analysis, I first consider the case when $\gamma = 1$. I then analyze how the results change with changes in γ in Section 4.

[13] Grossman and Helpman (2002) consider a similar two-stage bargaining model, where they assume that in the first stage, the negotiations between the parties can fail with some exogenous probability. See Inderst and Wey (2003), and de Fontenay and Gans (2005) for two more fully specified bargaining procedures which give rise to the Shapley value.

Specifically, S_i can try to approach B_j and B_i can try to approach S_j, where $i \neq j$. I assume that once the first-stage negotiations fail, B_i and S_i cannot meet to bargain again. If the second round of negotiations also fails, there are no more opportunities for trade.

The parties split the surplus from bargaining equally in both rounds of negotiations. In the first stage of the bargaining game, the parties negotiate knowing that if their negotiations fail, they may have the option of dealing with an alternative partner in the second stage. Therefore, the second stage of the bargaining game determines what the parties would get if the negotiations failed in the first stage of the bargaining game. An increase in the payoffs in the second stage of the bargaining game improves the bargaining positions of the parties in the first stage.

I now proceed to derive the pure-strategy symmetric subgame perfect Nash equilibria (SPNE) of the entire game by using backward induction.

3 Analysis

3.1 Bargaining Outcomes

I start the analysis by considering the case when $\gamma = 1$, which implies that for the buyers, the benefit from using any input variety in the unit interval is positive as long as the supplier of that input variety invests a positive amount. As γ increases, the buyers find it less and less appealing to use the inputs varieties that are different from their ideal variety. This is analyzed in Section 4.

To find the SPNE of the game specified in Section 2, I work backward from the determination of the output prices. Consider the final-stage subgame, where the buyers compete by choosing prices for given locations b_1 and $1 - b_2$. B_i maximizes $(p_i - c)\, D_i\,(p_1, p_2)$. The firms' demand functions can be determined by defining the indifferent consumer. After substituting for the demand functions in the profit functions of the firms, the Nash equilibrium in prices can be found. Solving the first-order conditions for profit maximization in prices simultaneously yields the result that buyer B_i's equilibrium price is equal to

$$p_i^*\,(b_i, b_j) = c + t\frac{\lambda\,(3 + b_i - b_j)}{3}, \tag{3}$$

where $\lambda = 1 - b_i - b_j$ stands for the degree of product differentiation in the downstream market. It is straightforward to check that the second-order condition is satisfied. The equilibrium price level is not affected by the suppliers' location and investment choices because the inputs bought affect the fixed costs of production only.

In the third stage of the game, the parties decide whether to trade after observing the location and investment choices made in the previous stages. I solve for the SPNE of the bargaining game specified in Section 2. The following lemma states the necessary and sufficient condition for trade to take place between B_i and S_i in the first stage of the bargaining game.

Lemma 1 *Given b_1, b_2, s_1, s_2, e_1 and e_2, consider the trade condition $TC\,(i)$ defined as*

$$e_i\left(1-(b_i-s_i)^2\right)-\frac{1}{2}e_j\left(1-(1-s_j-b_i)^2\right)\geq 0, \tag{4}$$

where $i,j=1,2$ and $i\neq j$.

(i) If $TC\,(i)$ holds for $i=1,2$, B_i and S_i trade one unit in the first stage of the bargaining game.

(ii) If $TC\,(i)$ holds but $TC\,(j)$ does not, B_i buys one unit from S_i in the first stage and B_j buys one unit from S_i in the second stage of the bargaining game.

(iii) If $TC\,(i)$ does not hold for $i=1,2$, B_i buys one unit from S_j, where $i\neq j$, in the second stage of the bargaining game.

Proof. All proofs are in the Appendix. ■

Since the suppliers' marginal cost of supplying additional units is zero, if the negotiations in the first stage of the bargaining game fail, the buyers can always find an alternative supplier to deal with in the second stage of the bargaining game. The trade condition $TC\,(i)$ given in (4) implies that B_i and S_i trade in the first stage of the bargaining game if the surplus created is nonnegative. The surplus created is the difference between the benefit of S_i's input to B_i and what B_i gets if the negotiations break down. Thus, trade takes place between B_i and S_i if it results in a higher value than B_i can get by trading with S_j. Since the benefit of an input from a specific supplier depends on that supplier's location and investment choices, (4) implies that B_i and S_i are more likely to trade as e_i increases or as S_i moves closer to B_i, and are less likely to trade as e_j increases or as S_j moves closer to B_i.

The trade condition $TC\,(i)$ also implies that the possibility that S_i may trade with B_j in the second stage does not affect the surplus created in the first stage. Since S_i is not capacity-constrained and can produce additional units at zero marginal cost, it can sell units to B_j in the second stage whether or not it trades with B_i in the first stage. Hence, the potential trade with B_j in the second stage does not have the impact of improving S_i's bargaining position in the first-stage negotiation with B_i.

Lemma 1 indicates that depending on whether $TC\,(i)$ holds for $i = 1, 2$, either the buyers deal with different suppliers or they both deal with the same supplier in equilibrium. In case (i), the buyers procure inputs from different suppliers in the first stage of the bargaining game. They receive their share of the surplus created and the payoff they would receive if the negotiations broke down. Hence, B_i and S_i's payoffs are

$$W_{B_i} = (p_i^* - c)\, D_i\,(p_1^*, p_2^*) + \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right) + \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right) \quad (5)$$

$$W_{S_i} = \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right) - \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right). \quad (6)$$

In case (ii), both buyers procure inputs from the same supplier in different stages of the bargaining game. This increases supplier S_i's payoff by $\frac{1}{2}e_i\left(1 - (1 - b_j - s_i)^2\right)$ and reduces supplier S_j's payoff to zero. Hence, the payoffs are

$$W_{B_i} = (p_i^* - c)\, D_i\,(p_1^*, p_2^*) + \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right) + \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right) \quad (7)$$

$$W_{B_j} = \left(p_j^* - c\right) D_j\,(p_1^*, p_2^*) + \frac{1}{2}e_i\left(1 - (1 - b_j - s_i)^2\right) \quad (8)$$

$$W_{S_i} = \frac{1}{2}e_i\left(2 - (b_i - s_i)^2 - (1 - b_j - s_i)^2\right) - \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right) \quad (9)$$

$$W_{S_j} = 0. \quad (10)$$

Finally, in case (iii), the buyers procure inputs from different suppliers in the second stage of the bargaining game. They receive their share of the surplus created. Hence, B_i and S_i's payoffs are

$$W_{B_i} = (p_i^* - c)\, D_i\,(p_1^*, p_2^*) + \frac{1}{2}e_j\left(1 - (1 - s_j - b_i)^2\right) \quad (11)$$

$$W_{S_i} = \frac{1}{2}e_i\left(1 - (1 - b_j - s_i)^2\right). \quad (12)$$

A comparison of (11) and (5) shows that having access to alternative suppliers helps the buyers. If $TC\,(i)$ holds, B_i's payoff is given by its share of the surplus created when it trades with S_i and the payoff it would receive if the negotiations broke down. If $TC\,(i)$ does not hold, B_i's payoff is determined by its share of the surplus created when it trades with S_j since the payoff it would receive if the negotiation with S_j broke down is zero. This is because trade takes place in the second stage of the bargaining game and, as stated in Section 2, if the second-stage negotiations fail, there are no more opportunities for trade.

3.2 Product and Investment Choices in the Upstream Market

In the second stage of the game outlined in Section 2, the suppliers simultaneously choose the input variety they would like to produce and their investment levels. S_i maximizes its earnings net of investment costs taking b_i, b_j, s_j and e_j as given. Its earnings in the input market for any given set of b_i, b_j, s_i, s_j, e_i and e_j depend on whether $TC(i)$ holds for $i = 1, 2$ and are as presented in Section 3.1.

Since the suppliers choose their locations after the buyers do, their decisions reflect the extent to which their production is customized to a specific buyer. Proposition 1 states that there is only one supplier choosing a positive amount of investment in equilibrium. The second supplier does not find it profitable to enter the market by investing a positive amount.

Proposition 1 *In equilibrium, both buyers purchase inputs from the same supplier, which chooses to produce an input type that is exactly in the middle between the two buyers' ideal input types. The supplier's investment level increases as the degree of product differentiation in the downstream market decreases.*

In the proof of Proposition 1, it is shown that even in cases when the second supplier could enter and produce a fully specialized input type, the buyers prefer to use a common input type that is not fully specialized for their purposes in order to benefit from the increased investment incentives of the single supplier. When both buyers purchase inputs from the same supplier, that supplier has increased incentives to invest because of the higher demand it faces.[14]

Suppose it is S_i that serves both buyers. Its payoff is given by (9). For given values of b_i and b_j, S_i's equilibrium choices of location and investment are

$$s_i^*(b_i, b_j) = b_i + \frac{\lambda}{2} \tag{13}$$

and

$$e_i^*(b_i, b_j) = 1 - \frac{\lambda^2}{4}. \tag{14}$$

The investment level increases as the buyers locate closer to each other because, given the supplier's optimal location choice, a decrease in the distance between the two buyers implies a

[14] To gain an understanding of the role investment plays in the model, it is instructive to consider briefly what the suppliers' location choices would be if they were not making any investment decisions. It is straightforward to conclude based on their payoff functions that if the suppliers chose locations only, they would prefer to produce fully specialized inputs. Hence, it is the interaction between the location and investment decisions of the suppliers that results in the emergence of a single supplier of a non-specialized input in the market.

decrease in the distance between the supplier and each of the buyers. This causes the buyers to receive a higher benefit from the inputs they obtain and the supplier to achieve a higher payoff.

3.3 Product Choices in the Downstream Market

In the first stage of the game, the buyers choose their locations to maximize their payoffs. This section examines how vertical interactions affect the product choices of buyers. Equation (1) implies that both buyers enter the market if $(p_i^* - c)\, D_i\,(p_1^*, p_2^*) \geq F_i$, where F_i is the net fixed cost of production after taking the cost reduction and input payment into account. In the following analysis, I restrict attention to the case when both buyers enter the market.[15]

The results in Section 3.2 indicate that for all $b_i, b_j \in [0, 1]$, there is only one supplier making a positive amount of investment in equilibrium. Hence, B_i and B_j maximize $W_{B_i} - K$ and $W_{B_j} - K$ respectively, where W_{B_i} and W_{B_j} are as defined in (7) and (8). Assuming S_i is the active supplier, $e_j = 0$ in equilibrium. This implies that the two payoff functions are symmetric.

Substituting for $s_i^*\,(b_i, b_j)$ and $e_i^*\,(b_i, b_j)$ in (7) and subtracting K gives

$$\pi_i = (p_i^* - c)\, D_i\,(p_1^*, p_2^*) - K + \frac{1}{2} e_i^*\,(b_i, b_j) \left(1 - (b_i - s_i^*\,(b_i, b_j))^2\right). \tag{15}$$

This expression illustrates the several effects that shape the location choices of the buyers. The first two terms, $(p_i^* - c)\, D_i\,(p_1^*, p_2^*) - K$, represent the profit level of B_i if it does not purchase any inputs. It is easy to verify that this is decreasing in b_i. In order to relax the price competition between themselves, the firms would prefer to locate as far from each other as possible. I refer to this as the *competition effect*.

The third term, $\frac{1}{2} e_i^*\,(b_i, b_j) \left(1 - (b_i - s_i^*\,(b_i, b_j))^2\right)$, represents the benefit B_i gets from using S_i's input. This benefit has two components. First, it depends on the level of investment made by S_i. Equation (14) implies that $e_i^*\,(b_i, b_j)$ is increasing in b_i. An increase in b_i results in a decrease in the degree of product differentiation in the downstream market. The supplier invests more when the buyers are located closer together because the value created from the use of the inputs is higher. Hence, in order to increase the investment incentives of their suppliers, the buyers prefer to locate closer to each other. This is the *investment effect*.

[15] After solving the buyers' maximization problem, I specify the parameter values for which this assumption will hold in the proof of Proposition 2.

The benefit B_i gets from using S_i's input also depends on how suitable S_i's input is for B_i's purposes, which is represented by $\left(1 - \left(b_i - s_i^*\left(b_i, b_j\right)\right)^2\right)$. It is easy to verify that this term is increasing in b_i. This *specificity effect* reflects the fact that the buyers always like to locate close to the suppliers they trade with. They know that for given values of b_i and b_j, the suppliers always choose locations between them. The specificity effect indicates that for a given value of b_j, as b_i increases, the distance between B_i and S_i decreases and the buyer benefits from using an input which is more suitable for its purposes.

The location choice of B_i depends on which of the three effects dominate. After substituting the exact expressions for $s_i^*\left(b_i, b_j\right)$ and $e_i^*\left(b_i, b_j\right)$ in (15), B_i solves

$$\max_{b_i} \ \pi_i = t\frac{\lambda\left(3 + b_i - b_j\right)^2}{18} + \frac{\left(4 - \lambda^2\right)^2}{32} - K \tag{16}$$

taking b_j as given. Given the symmetry between the firms, I focus on symmetric equilibrium. Note that when $t = 0$, the optimal product choices are $b_i^* = b_j^* = b^* = \frac{1}{2}$. That is, when the consumers are indifferent about which variety they consume, the firms prefer to locate in the middle. In the proof of Proposition 2, it is shown that as t increases, b^* monotonically decreases. Hence, the following result can be stated.

Proposition 2 *The buyers' symmetric equilibrium location choice is decreasing in t. For sufficiently low values of t, the investment and specificity effects dominate the competition effect, and the buyers choose to be less than maximally differentiated.*

A low t value implies that even with maximum differentiation, the buyers face intense competition because the consumers care less about consuming a variety that is different from their ideal one. Proposition 2 states that as the measure of consumer loyalty, t, decreases, the buyers have increased incentives to agglomerate in the middle. As t decreases and the buyers cannot effectively decrease the rivalry between them by locating further away from each other, the investment and specificity effects start to play relatively more important roles in their location decisions. Hence, they may choose less than maximum differentiation both in order to increase the investment incentives of the suppliers and to use more suitable inputs.

The analysis reveals that once the classic Hotelling model with quadratic costs is extended to take into account the dynamics of the interactions between upstream suppliers and downstream buyers, buyers may choose intermediate locations in the product space in order to be close to the only supplier in the market which is located in the middle.

4 Changes in Relative Importance of Specificity

In the analysis so far, γ was restricted to 1. A question that arises from the analysis in Section 3 is how the magnitude of γ affects the equilibrium outcome. In this section, I explore whether there can be a SPNE of the entire game where both suppliers make positive investment choices.

It is straightforward to show that the trade condition given in (4) is modified in the following way.

$$e_i\left(1-\gamma\left(b_i-s_i\right)^2\right)-\max\left\{0,\frac{1}{2}e_j\left(1-\gamma\left(1-s_j-b_i\right)^2\right)\right\}\geq 0. \tag{17}$$

This is because while negotiating with S_i, B_i may have the option to trade with S_j in the second stage of the bargaining process if and only if the benefit it receives from using S_j's input, $e_j\left(1-\gamma\left(1-s_j-b_i\right)^2\right)$, is positive.

In the second stage of the game, for a given value of γ, either one supplier or both suppliers make positive investments depending on the values of b_i and b_j. The following proposition states the SPNE outcome in the second stage of the game for different λ values.

Proposition 3 *In the second stage of the game, there exist two critical λ values, $0<\widetilde{\lambda}(\gamma)<\widehat{\lambda}(\gamma)$, such that:*

(i) For $\lambda<\widetilde{\lambda}(\gamma)$, only one supplier chooses a positive investment level and locates between the two buyers.

(ii) For $\widetilde{\lambda}(\gamma)\leq\lambda<\widehat{\lambda}(\gamma)$, there are two types of equilibria. In one of them, only one supplier chooses a positive investment level and locates between the two buyers. In the other one, both suppliers choose positive investment levels. S_i serves B_i and chooses $s_i=b_i$.

(iii) For $\lambda\geq\widehat{\lambda}(\gamma)$, there are two types of equilibria, both of which have both suppliers choosing positive investment levels and producing fully specialized inputs. In one of them, S_i serves B_i and in the other one, S_i serves B_j for $i=1,2$ and $i\neq j$.

Proposition 3 states that for a given γ value, if the degree of product differentiation in the downstream market is sufficiently small, there will be only one supplier choosing a positive investment level and serving both buyers. If the degree of product differentiation is sufficiently large, there exist SPNE outcomes where both suppliers make positive investment levels and choose to produce fully specialized inputs. This is because as the degree of product differentiation increases, it becomes too costly to use non-specialized inputs even if doing so results in higher investment incentives. The proposition also states that there exists a range of λ values for which there is a multiplicity of equilibria in the second stage of the game.

Corollary 1 *As γ increases, it becomes less desirable to have one common supplier and more desirable to have two fully specialized suppliers.*

In the proof of Corollary 1, I show that the critical values $\widehat{\lambda}(\gamma)$ and $\widetilde{\lambda}(\gamma)$ given in Proposition 3 are decreasing in γ. As γ increases, the buyers get higher disutility from using an input type that is not perfectly suited to their needs. This is the reason the range of λ values for which only one supplier serves both buyers in equilibrium gets smaller. On the other hand, for sufficiently small values of γ such that $\widetilde{\lambda}(\gamma) > 1$, there exists a unique SPNE where a single supplier serves both buyers. The analysis in Section 3 implies that the critical γ value such that $\widetilde{\lambda}(\gamma) = 1$ must be greater than 1.

I next show that whenever λ is high enough for both suppliers to make positive investments, the benefit the buyers get from using the input of the alternative supplier is zero. That is, they do not have the option of obtaining inputs from the alternative supplier if the negotiation in the first stage fails.

Lemma 2 *For all λ values such that there are two suppliers making positive investments in equilibrium (i.e., for $\widetilde{\lambda}(\gamma) \leq \lambda \leq 1$), the buyers do not benefit from the inputs of the alternative suppliers.*

In the proof of Lemma 2, it is shown that the range of λ values such that the buyers benefit from having access to alternative suppliers is smaller than the range of λ values such that a single supplier serves both buyers. This implies that whenever the buyers receive a positive benefit from using the input of the alternative supplier located at distance λ from them (i.e., where the rival buyer is located), one of the suppliers has an incentive to deviate and locate such that it serves both buyers. This is because if a supplier deviates to serve both buyers, it locates between the two buyers. Hence, the distance between the supplier and each of the buyers is less than λ, the distance between the two buyers. This guarantees that both buyers receive a net benefit from using the deviating supplier's inputs and implies that the deviation will be profitable.

An increase in the number of input varieties in the market can have two kinds of effect on the buyers' payoff. The direct effect is that with more variety, each buyer may be able to find an input type which is more suited to its needs. The indirect effect is that having an alternative supplier to deal with increases the buyers' bargaining power. Corollary **??** implies that even if the number of varieties in the market increases, the buyers do not benefit from having access to an alternative supplier in the bargaining process. The existence of two varieties helps the buyers in getting more specialized inputs only. Hence, competition between the suppliers never arises in the sense that the suppliers are never harmed in the bargaining process by the existence of an alternative supplier in the market.

I now turn to the equilibrium of the entire game. The following proposition states that there exist parameter values for which both suppliers make positive investments in equilibrium.

Proposition 4 *(i) If $\widetilde{\lambda}(\gamma) > 1$, then the unique type of equilibrium involves one supplier serving both buyers. The supplier locates exactly in the middle between the two buyers and the buyers choose intermediate locations for sufficiently small t values. (ii) If $\widetilde{\lambda}(\gamma) < 1$, there exist values of t and γ such that both suppliers making positive investments and producing fully specialized inputs is a SPNE. (iii) There is no SPNE where both of the suppliers make positive investments and the buyers choose intermediate locations.*

Part (i) of Proposition 4 extends the result from Section 3. For values of γ such that $\widetilde{\lambda}(\gamma) > 1$, one supplier serving both buyers is the SPNE. Part (ii) of Proposition 4 states that for $\widetilde{\lambda}(\gamma) < 1$, the buyers may prefer to locate such that both suppliers make positive investments in equilibrium. Finally, part (iii) of Proposition 4 implies that one would expect to see the buyers choosing intermediate locations only in cases when both buyers are served by the same supplier. In an industry where multiple suppliers serve downstream buyers by producing specialized inputs, one would not expect to see less than maximum differentiation in the downstream market. In the subgames where there will be two suppliers in the market, the buyers choose maximum differentiation because they do not benefit from the inputs of the alternative supplier.

5 Conclusion

Firms often differentiate their products through the use of inputs that are differentiated or specialized. This implies that product choices made in the downstream markets cannot be analyzed in isolation from the product choices made in the upstream markets. The goal of this paper has been to explore how the demand for specific investments in the production of inputs may affect product variety in a bilateral duopolistic industry. The degree of specificity is endogenously determined in the model through the product choices made by all the firms in the market. This is because if the buyers produce more similar products, the suppliers' inputs become less specialized.

I have specifically studied when suppliers choose to produce specialized inputs and when buyers may choose to be less than maximally differentiated. The analysis shows that two things play critical roles in the decisions of the suppliers: (i) the degree of product differentiation in the downstream market, and (ii) the importance of having specialized inputs. The benefit from a specific input type depends on how well suited the input is for the needs of the buyer and how much effort the supplier has put into its development. Under incomplete contracts, there may be a trade-off between these two. This implies that if the suppliers have no capacity constraints and if the buyers are flexible in using inputs that are not perfectly suited for their purposes, only one supplier serves the downstream market in equilibrium. The supplier chooses to locate between the two buyers. With only a single supplier in the market, the buyers do not have access to specialized inputs, but the supplier invests more in the development of the input.

As the importance of having specific inputs increases, fully specialized suppliers may emerge. It becomes unprofitable for the buyers to locate such that only one supplier invests in equilibrium because this would require them to approach each other too much. Instead, they choose to be maximally differentiated. Hence, the buyers choose intermediate locations only in cases when they want to encourage entry by a single supplier.

An important way in which the current analysis can be extended is by considering the case where inputs can determine both the marginal and fixed costs of production. If the suppliers were to invest to decrease the buyers' marginal cost of production, a change in the locations of the buyers would affect the suppliers' payoffs through two different channels. The suppliers' payoffs and investment incentives would change because of a change in both the profits made in the downstream market and the benefits obtained from the inputs. As in McLaren (2000), the current set-up has allowed me to differentiate between these two effects and concentrate on the latter one. With the insights from the current analysis, it would be worthwhile to analyze the case when inputs determine the marginal cost of production.

References

[1] Aghion, P. and J. Tirole. 1994. "The Management of Innovation," *The Quarterly Journal of Economics*, 109(4), 1185-209.

[2] Belleflamme, P. and E. Toulemonde. 2003. "Product Differentiation in Successive Vertical Oligopolies," *Canadian Journal of Economics*, 36(3), 523-45.

[3] Choi, J. P. and S.-S. Yi. 2000. "Vertical Foreclosure with the Choice of Input Specifications," *Rand Journal of Economics*, 31(4), 717-43.

[4] Church, J. and N. Gandal. 2000. "Systems Competition, Vertical Merger and Foreclosure," *Journal of Economics and Management Strategy*, 9, 25-51.

[5] Columbus Dispatch, The. 2006. "Coal's Dirty Part: Thousands of Acres of Ohio Remain Polluted, Dangerous," December 4.

[6] D'Aspremont, C., J. J. Gabszewicz, and J.-F. Thisse. 1979. "On Hotelling's Stability in Competition," *Econometrica*, 47, 1045-50.

[7] De Fontenay, C. C. and J. S. Gans. 2005. "Vertical Integration in the Presence of Upstream Competition," *Rand Journal of Economics*, 36(3), 544-572.

[8] Farrell, J. and N. T. Gallini. 1988. "Second Sourcing as a Commitment: Monopoly Incentives to Attract Competition," *Quarterly Journal of Economics*, 103(4), 673-94.

[9] Friedman, J. M. and J.-F. Thisse. 1993. "Partial Collusion Fosters Minimum Product Differentiation," *Rand Journal of Economics*, 24, 631-45.

[10] Fujita, M. and J.-F. Thisse. 1996. "Economics of Agglomeration," *Journal of the Japanese and International Economics*, 10, 339-78.

[11] Ghosh, A. and H. Morita. 2006. "Platform Sharing in a Differentiated Duopoly," *Journal of Economics and Management Strategy*, 15, 397-429.

[12] Grossman, G. M. and E. Helpman. 2002. "Integration versus Outsourcing in Industry Equilibrium," *Quarterly Journal of Economics*, 117(1), 85-120.

[13] Helper, S. and D. I. Levine. 1992. "Long-Term Supplier Relations and Product-Market Structure," *Journal of Law, Economics and Organization*, 8(3), 561-581.

[14] Hotelling, H. 1929. "Stability in Competition," *Economic Journal*, 39, 41-57.

[15] Inderst, R. and C. Wey. 2003. "Bargaining, Mergers, and Technology Choice in Bilaterally Oligopolistic Industries," *Rand Journal of Economics*, 34(1), 1-19.

[16] Krishnan, V. and K. T. Ulrich. 2001. "Product Development Decisions: A Review of the Literature," *Management Science*, 47(1), 1-21.

[17] Marshall, A. 1920. *Principles of Economics*, 8th edition, Macmillan and Co., London.

[18] McLaren, J. 2000. "'Globalization' and Vertical Structure," *American Economic Review*, 90(5), 1239-54.

[19] De Palma, A., V. Ginsburgh, Y. Y. Papageorgiou, J.-F. Thisse. 1985. "The Principle of Minimum Differentiation Holds Under Sufficient Heterogeneity," *Econometrica*, 53, 767-81.

[20] Pepall, L. and G. Norman. 2001. "Product Differentiation and Upstream-Downstream Relations," *Journal of Economics and Management Strategy*, 10, 201-33.

[21] Riordan, M. H. and O. E. Williamson. 1985. "Asset Specificity and Economic Organization," *International Journal of Industrial Organization*, 3, 365-78.

[22] Robertson, D. and K. Ulrich. 1998. "Planning for Product Platforms," *Sloan Management Review*, 39(4), 19-31.

[23] Shepard, A. 1987. "Licensing to Enhance Demand for New Technologies," *Rand Journal of Economics*, 18(3), 360-68.

[24] Thisse, J.-F. and G. Norman. 1995. *The Economics of Product Differentiation*, Edward Elgar, Aldershot, UK.

Appendix

This Appendix contains the proofs of Corollary 1, Lemmas 1-2, and Propositions 1-4.

1 Proof of Lemma 1

At the end of the first stage, there are 3 types of subgames: 1) Both pairs have traded 1 unit in the first stage, 2) one of the pairs has traded 1 unit and the other pair has traded 0 units in the first stage, and 3) both pairs have traded 0 units in the first stage. In a subgame of type 1, no more trade takes place in the second stage because each buyer can use at most 1 unit. In the other two types of subgames, the buyers with 0 units can obtain inputs in the second stage. Since the suppliers are not capacity-constrained, their ability to do so does not depend on whether or not trade took place between the other pair in the first stage.

Consider the negotiation between B_i and S_j in the second stage. The gains from trade are $e_j\left(1-(1-b_i-s_j)^2\right)$, which is the benefit B_i gets from using one unit of the input produced by S_j. Hence, B_i buys an input from S_j in the second stage if it has 0 units at the end of the first stage and if $e_j > 0$. If trade take place, B_i and S_j receive the sum of their share of this amount and their outside option. B_i's outside option is determined by the amount it would make in the downstream market. S_j's outside option is 0 since it cannot find another trading partner after the second stage of the bargaining game is over.

In the first stage of the bargaining game, $B_i - S_i$ and $B_j - S_j$ bargain simultaneously. To determine mutual best responses, consider the negotiation between B_i and S_i. Suppose B_j and S_j do not trade. S_i can negotiate with B_j in the second stage whether or not it trades with B_i in the first stage. If $e_i\left(1-(1-b_j-s_i)^2\right) > 0$, it sells 1 unit to B_j and earns $\frac{1}{2}e_i\left(1-(1-b_j-s_i)^2\right)$. If B_i and S_i do not trade, B_i can negotiate with S_j. If it trades with S_j, the net benefit it gets is $\frac{1}{2}e_j\left(1-(1-s_j-b_i)^2\right)$. Hence, B_i and S_i's joint profits in case of not trading are

$$(p_i^* - c)\, D_i\left(p_1^*, p_2^*\right) + \frac{1}{2}e_j\left(1-(1-s_j-b_i)^2\right) + \frac{1}{2}e_i\left(1-(1-b_j-s_i)^2\right). \tag{A.1}$$

If B_i and S_i trade 1 unit, their joint profits are

$$(p_i^* - c)\, D_i\left(p_1^*, p_2^*\right) + e_i\left(1-(b_i-s_i)^2\right) + \frac{1}{2}e_i\left(1-(1-b_j-s_i)^2\right). \tag{A.2}$$

Comparing the joint profits reveals that B_i and S_i trade 1 unit if (4) holds and trade 0 units if it does not hold.

A similar analysis reveals that the best response of B_i and S_i in the case when B_j and S_j trade unit is determined by (4) also. Hence, for every possible strategy of B_j and S_j, B_i and S_i trade if (4) holds and do not trade if it does not.

Labelling the inequality in (4) as $TC(i)$, we get the following cases. If both $TC(i)$ and $TC(j)$ hold, both pairs trade in the first stage of the bargaining game. If only $TC(i)$ holds, B_j and S_j do not trade in the first stage of the bargaining game. If neither $TC(i)$ nor $TC(j)$ holds, none of the pairs trades in the first stage of the bargaining game. The firms' payoffs in the different cases are given in Section 3.1.

2 Proof of Proposition 1

The proof proceeds in two steps.

(i) I first show that neither supplier has an incentive to deviate from the equilibrium stated in Proposition 1. In equilibrium, supplier S_i earns the maximum possible profit by being the sole supplier of inputs to both buyers. Hence, it cannot do any better by deviating. S_j, on the other hand, makes 0. It is necessary to check whether it can earn a positive profit by setting s_j and e_j such that it trades with B_j only, with B_i only, or with both of the buyers.

If it trades with B_j, its payoff is equal to $W_{S_j} - \frac{e_j^2}{2}$, where W_{S_j} is given by (6) after switching i and j. The unconstrained optimum is $s_j = b_j$ and $e_j = \frac{1}{2}$. For B_j to prefer to trade with S_j, $TC(j)$ must be holding. Given the equilibrium strategy of S_i, this condition is satisfied. Substituting for the relevant choices of s_i, s_j, b_i and b_j yields $W_{S_j} = \frac{1}{8} - \frac{(4-\lambda^2)^2}{64}$, which is negative for all $\lambda \in [0,1]$. Hence, this is not a profitable deviation.

If it trades with B_i, it makes $W_{S_j} - \frac{e_j^2}{2}$, where W_{S_j} is given by (12) after switching i and j. The unconstrained optimum is $s_j = 1 - b_i$ and $e_j = \frac{1}{2}$. For B_i to prefer to buy from S_j instead of from S_i, it must be the case that $TC(i)$ is violated. Given the equilibrium strategy of S_i, $TC(i)$ is satisfied at the unconstrained optimum stated above. Supplier S_j can choose s_j and e_j such that it is just violated. The constrained optimal choices are $s_j = 1 - b_i$ and $e_j = \frac{(4-\lambda^2)^2}{8}$. Evaluating S_j's profit level at these choices gives

$$\left(\frac{(4-\lambda^2)^2}{16}\right)\left(1 - \frac{(4-\lambda^2)^2}{8}\right). \tag{A.3}$$

Again, since this is negative for all $\lambda \in [0,1]$, this is not a profitable deviation.

Finally, S_j can set s_j and e_j such that both buyers prefer to trade with it. This implies that its choices must violate $TC(i)$ and satisfy $TC(j)$. S_j's payoff is equal to $W_{S_j} - \frac{e_j^2}{2}$, where W_{S_j} is given by (9) after switching i and j. If $TC\left(i\right)$ and $TC\left(j\right)$ are evaluated at the unconstrained optimum of S_j's payoff function, both of them are satisfied, which implies that B_i prefers to trade with S_i instead of S_j. Hence, consider values of s_i and e_i such that $TC\left(i\right)$ is just violated. Note that if there is not a profitable deviation that satisfies only one of the constraints, there cannot be a profitable deviation that satisfies both constraints. Setting $TC\left(i\right)=0$ and solving for e_j yields $e_j=\frac{\left(4-\lambda^2\right)^2}{8\left(1-\left(1-b_i-\widetilde{s}_j\right)^2\right)}$, where $\widetilde{s}_j$ represents the location choice that solves the constrained optimization problem. These choices of s_j and e_j constitute a profitable deviation if they result in a positive profit level. Substituting for this expression for e_j in S_j's payoff function gives

$$\frac{\left(4-\lambda^2\right)^2}{16}\left[\frac{\left(1-\left(b_j-\widetilde{s}_j\right)^2\right)}{\left(1-\left(1-b_i-\widetilde{s}_j\right)^2\right)}+\frac{3}{4}-\frac{\left(4-\lambda^2\right)^2}{8\left(1-\left(1-b_i-\widetilde{s}_j\right)^2\right)^2}\right]. \tag{A.4}$$

Redefining S_j's choice variable as $\sigma=1-b_i-s_j$ and letting $\widetilde{\sigma}=1-b_i-\widetilde{s}_j$ yields

$$\frac{\left(4-\lambda^2\right)^2}{16}\left[\frac{\left(1-\left(\widetilde{\sigma}-\lambda\right)^2\right)}{\left(1-\widetilde{\sigma}^2\right)}+\frac{3}{4}-\frac{\left(4-\lambda^2\right)^2}{8\left(1-\widetilde{\sigma}^2\right)^2}\right]. \tag{A.5}$$

It is easy to verify graphically that this expression is negative for $\widetilde{\sigma},\lambda\in[0,1]$.

Hence, there are no profitable deviations. The location and investment choices stated in Proposition 1 constitute an equilibrium.

(ii) I next show that the equilibrium stated in Proposition 1 is unique. To do this, all other possible equilibria have to be eliminated.

I start by establishing that in equilibrium, $TC\left(i\right)$ for $i=1,2$ never bind. To prove by contradiction, suppose not and suppose it is $TC\left(1\right)$ that binds in equilibrium. Then S_2 can increase its investment by ε and cause a discontinuous upward jump in its payoff function. Since the cost of investment is continuous in the level of investment, there always exists a sufficiently small investment level that makes such a deviation profitable.

This result implies that I can focus on the interior solutions to the suppliers' optimization problem as candidate equilibria. There are three types of candidate equilibria. I have already shown that one of them is an equilibrium. To eliminate the other two, it is sufficient to find a profitable deviation in each case.

Consider the candidate equilibrium where S_i trades with B_i for $i=1,2$ in the first stage of the bargaining game. The optimum choices are $s_i=b_i$ and $e_i=\frac{1}{2}$. Note that these location and investment choices imply that $TC\left(i\right)$ is satisfied. Each supplier makes $\frac{1-\left(1-\lambda^2\right)}{8}$.

Now suppose S_1 plays its equilibrium strategy and S_2 deviates by choosing s_2 and e_2 such that it serves both buyers. It must be the case that $TC(1)$ is violated and $TC(2)$ holds. S_2 cannot set s_2 and e_2 at their unconstrained maximum levels since doing so does not violate $TC(1)$. If it chooses s_2 and e_2 such that $TC(1)$ is just violated, it must set $e_2 = \frac{1}{\left(1-(1-b_1-\widehat{s}_2)^2\right)}$, where $\widehat{s}_2$ represents the constrained optimal choice for location. S_2 makes

$$\frac{\left(1-(b_2-\widehat{s}_2)^2\right)}{2\left(1-(1-b_1-\widehat{s}_2)^2\right)}+\frac{1}{2}-\frac{\left(1-\lambda^2\right)}{8}-\frac{1}{2\left(1-(1-b_1-\widehat{s}_2)^2\right)^2}. \tag{A.6}$$

Again, letting $\widehat{\sigma} = 1 - b_1 - \widehat{s}_2$ yields

$$\frac{\left(1-(\widehat{\sigma}-\lambda)^2\right)}{2\left(1-\widehat{\sigma}^2\right)}+\frac{1}{2}-\frac{\left(1-\lambda^2\right)}{8}-\frac{1}{2\left(1-\widehat{\sigma}^2\right)^2}. \tag{A.7}$$

Since it is sufficient to find only one profitable deviation, let us set $\widehat{\sigma} = \frac{2}{5}$. The payoff function given in (A.6) always yields a higher value than $\frac{1-\left(1-\lambda^2\right)}{8}$ for all $\lambda \in [0,1]$.

Now consider the case when S_i trades with B_j for $i, j = 1, 2$ and $i \neq j$ in the second stage of the bargaining game. The optimum location and investment choices are $s_i = 1 - b_j$ and $e_i = \frac{1}{2}$. Each firm makes $\frac{1}{8}$. Suppose S_1 plays its equilibrium strategy and S_2 deviates by choosing s_2 and e_2 such that it serves both buyers. It sets $s_2 = b_2 + \frac{\lambda}{2}$ and $e_2 = \frac{4-\lambda^2}{4}$. Note that $TC(1)$ is violated and $TC(2)$ is satisfied at these location and investment choices. S_2's payoff if it sells to both buyers is $\frac{\left(4-\lambda^2\right)^2}{32} - \frac{1}{8}$. This is larger than $\frac{1}{8}$ for all $\lambda \in [0,1]$.

Hence, the equilibrium where S_i sells to both buyers and S_j sets $e_j = 0$ is the unique equilibrium.

3 Proof of Proposition 2

It can be shown that as t increases from 0, b^* changes in a monotonic way by using the implicit function theorem. Let $G(b_i, b_j; t)$ stand for the first derivative of $\pi_i = W_{B_i} - K$ with respect to b_i. It is equal to

$$\frac{(3-b_i-b_j)\left(1-(b_i-b_j)^2\right)}{8}-\frac{t(3+b_i-b_j)(1+3b_i+b_j)}{18}. \tag{A.8}$$

This expression is equal to 0 at an interior solution. To solve for the symmetric equilibrium, let $b_i = b_j = b^*$. The implicit function theorem states

$$\frac{\partial b^*}{\partial t} = -\frac{\partial G(b_i, b_j; t)/\partial t}{\partial G(b_i, b_j; t)/\partial b}, \tag{A.9}$$

where the right hand side is evaluated at b^*. Since the second-order condition is negative, the sign of $\partial b^*/\partial t$ is equal to the sign of the numerator. The numerator is

$$\frac{\partial G(b_i, b_j; t)}{\partial t} = \frac{-(3 + b_i - b_j)(1 + 3b_i + b_j)}{18} < 0. \tag{A.10}$$

Hence, b^* is a decreasing function of t. As t increases from 0, b^* decreases from 1/2 to 0 in a continuous fashion.

To determine the exact parameter values for which the buyers will be less than maximally differentiated, note that the analysis so far is based on the assumptions that K is low enough for both buyers to produce positive quantities, and that K is higher than the cost reduction obtained from the inputs provided by the suppliers. Hence, it remains to show that there exist t values for which both of these assumptions are satisfied and the buyers choose intermediate locations. The two assumptions imply that

$$\frac{\left(4-\lambda^2\right)^2}{16} \leq K \leq t\frac{\lambda(3 + b_i - b_j)^2}{18} + \frac{\left(4-\lambda^2\right)^2}{32}, \tag{A.11}$$

where the first term stands for the benefit from an input and the last two terms represent the profits in the product market and the net benefit from an input (after making the input payment) respectively. It is straightforward to verify that b^* is positive for $t < 2.25$. Evaluating (A.11) at b^* and numerically solving for t reveals that the lower limit on K is smaller than the upper limit for $t > 1.13$, which implies that for these t values, there exist K values which satisfy the inequalities given in (A.11). Hence, for $1.13 < t < 2.25$, the buyers choose intermediate locations.[16]

4 Proof of Proposition 3

There are three types of candidate equilibria. In the following claims, I consider each in turn.

Claim 1 *If* $\lambda < \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$, *$S_i$ locates in the middle between the two buyers and sells inputs to both.* $e_i = 1 - \frac{\gamma\lambda^2}{4}$ *while* $e_j = 0$.

Proof. The value of the input to both buyers is positive if $\lambda < \frac{2}{\sqrt{\gamma}}$. This condition is automatically satisfied for all $\lambda < \widehat{\lambda}$. Can supplier S_i do better by deviating? Deviating

[16] Obviously, the exact values given for t are highly sensitive to the model specification. It can be easily verified that the range of t values for which the buyers choose intermediate locations gets larger with sufficiently small downward shifts in the investment cost function or sufficiently small upward shifts in the input benefit function such that there is still only one supplier in the market in equilibrium.

and locating at the same point as one of the buyers may be preferable for sufficiently large λ values. If S_i serves one of the buyers only, its profit-maximizing location and investment choices are $s_i = b_i$ and $e_i = \frac{1}{2}$. Such a deviation may be more profitable only in the case when B_j does not find it profitable to use S_i's input when S_i locates at $s_i = b_i$. Otherwise, the supplier's profit-maximizing options are as given above.

The payoff to S_i from deviating and serving one of the buyers only is $\frac{1}{8}$. Comparing this with S_i's equilibrium payoff yields that such a deviation is profitable if and only if $\lambda > \sqrt{\frac{2}{\gamma}}$. Hence, for $\lambda < \widehat{\lambda}$, S_i does not have a profitable deviation.

Consider now whether supplier S_j can do better by deviating. It makes 0 in equilibrium. It is necessary to check whether it can earn a positive profit by setting s_j and e_j such that it trades with B_j only, with B_i only, or with both of the buyers. If it trades with B_j, it makes

$$\frac{1}{2}\left[e_j\left(1-\gamma\left(b_j-s_j\right)^2\right)-\frac{1}{2}e_i\left(1-\gamma\left(1-b_j-s_i\right)^2\right)\right]-\frac{e_j^2}{2}. \tag{A.12}$$

The unconstrained optimum is $s_j = b_j$ and $e_j = \frac{1}{2}$. For B_j to prefer to trade with S_j, $TC\left(j\right)$ must be holding. Given the equilibrium strategy of S_i, this condition is satisfied. Substituting for the relevant choices of s_i, s_j, e_i and e_j yields $\frac{1}{8} - \frac{\left(4-\gamma\lambda^2\right)^2}{64}$, which is negative for $\lambda < \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$. Hence, this is not a profitable deviation.

If it trades with B_i, it makes

$$\frac{1}{2}e_j\left(1-\gamma\left(1-b_i-s_j\right)^2\right)-\frac{e_j^2}{2}. \tag{A.13}$$

The location and investment choices that maximize this payoff function are $s_j = 1 - b_i$ and $e_j = \frac{1}{2}$. For B_i to prefer to buy from S_j instead of from S_i, it must be the case that $TC\left(i\right)$ is violated. Given the equilibrium strategy of S_i, $TC\left(i\right)$ holds at the unconstrained optimum stated above for $\lambda < \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$. Supplier S_j can choose s_j and e_j such that $TC\left(i\right)$ is just violated. The constrained optimal choices are $s_j = 1 - b_i$ and $e_j = \frac{\left(4-\gamma\lambda^2\right)^2}{8}$. Evaluating S_j's profit level at these choices yields

$$\left(\frac{\left(4-\gamma\lambda^2\right)^2}{16}\right)\left(\frac{8-\left(4-\gamma\lambda^2\right)^2}{8}\right). \tag{A.14}$$

Again, since this expression is negative for $\lambda < \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$, this is not a profitable deviation.

Finally, S_j can set s_j and e_j such that both buyers prefer to trade with it. This implies that its choices must violate $TC\left(i\right)$ and satisfy $TC\left(j\right)$. S_j's payoff is given by

$$\frac{1}{2}\left[e_j\left(2-\gamma\left(b_j-s_j\right)^2-\gamma\left(1-b_i-s_j\right)^2\right)-\frac{1}{2}e_i\left(1-\gamma\left(1-b_j-s_i\right)^2\right)\right]-\frac{e_j^2}{2}. \tag{A.15}$$

If $TC(i)$ and $TC(j)$ are evaluated at the unconstrained optimum of this payoff function, both of them are satisfied, which implies that B_i prefers to trade with S_i instead of S_j. Hence, consider values of s_i and e_i such that $TC(i)$ is just violated. Note that if there is not a profitable deviation that satisfies only one of the constraints, there cannot be a profitable deviation that satisfies both of the constraints. Setting $TC(i) = 0$ and solving for e_j yields $e_j = \frac{(4-\gamma\lambda^2)^2}{8\left(1-\gamma(1-b_i-\widetilde{s}_j)^2\right)}$, where $\widetilde{s}_j$ represents the location choice that solves the constrained optimization problem. These choices of s_j and e_j constitute a profitable deviation if they result in a positive profit level. Substituting for the expressions for e_j, e_i and s_i in S_j's payoff function yields

$$\frac{(4-\gamma\lambda^2)^2}{16}\left[\frac{\left(1-\gamma(b_j-\widetilde{s}_j)^2\right)}{\left(1-\gamma(1-b_i-\widetilde{s}_j)^2\right)}+\frac{3}{4}-\frac{(4-\gamma\lambda^2)^2}{8\left(1-\gamma(1-b_i-\widetilde{s}_j)^2\right)^2}\right]. \tag{A.16}$$

Redefining S_j's choice variable as $\sigma = 1-b_i-s_j$ and letting $\widetilde{\sigma} = 1-b_i-\widetilde{s}_j$ generates

$$\frac{(4-\gamma\lambda^2)^2}{16}\left[\frac{\left(1-\gamma\lambda^2\left(1-\frac{\widetilde{\sigma}}{\lambda}\right)^2\right)}{\left(1-\gamma\lambda^2\left(\frac{\widetilde{\sigma}}{\lambda}\right)^2\right)}+\frac{3}{4}-\frac{(4-\gamma\lambda^2)^2}{8\left(1-\gamma\lambda^2\left(\frac{\widetilde{\sigma}}{\lambda}\right)^2\right)^2}\right], \tag{A.17}$$

where $\frac{\widetilde{\sigma}}{\lambda}$ represents the ratio of the distance between S_j and B_i to the distance between B_i and B_j. Since S_j deviates to serve both buyers, its optimal location is between the two buyers. This implies that $\frac{\widetilde{\sigma}}{\lambda}$ ranges between 0 and 1. Substituting for $\gamma\lambda^2 = \theta$ yields

$$\frac{(4-\theta)^2}{16}\left[\frac{\left(1-\theta\left(1-\frac{\widetilde{\sigma}}{\lambda}\right)^2\right)}{\left(1-\theta\left(\frac{\widetilde{\sigma}}{\lambda}\right)^2\right)}+\frac{3}{4}-\frac{(4-\theta)^2}{8\left(1-\theta\left(\frac{\widetilde{\sigma}}{\lambda}\right)^2\right)^2}\right], \tag{A.18}$$

To prove the claim, it is necessary to check whether this expression is negative for $\frac{\widetilde{\sigma}}{\lambda} \in [0,1]$ and $\theta \in \left[0, 4-2\sqrt{2}\right)$. It is easy to verify graphically that this is the case. Hence, for $\lambda < \widehat{\lambda}$, there are no profitable deviations from the equilibrium where S_i sells inputs to both buyers by choosing $s_i = b_i + \frac{\lambda}{2}$ and $e_i = 1 - \frac{\gamma\lambda^2}{4}$, and S_j invests zero. ■

Claim 2 *If $\lambda > \widetilde{\lambda}$ where $0 < \widetilde{\lambda} < \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$, S_i trades with B_i for $i = 1,2$ in the first stage of the bargaining game. The optimal choices are $s_i = b_i$ and $e_i = \frac{1}{2}$.*

Proof. Since $TC(i)$ is satisfied at the equilibrium location and investment choices, each supplier makes $\frac{1}{8} - \max\left\{0, \frac{(1-\gamma\lambda^2)}{8}\right\}$ in equilibrium. To see that there are no profitable deviations for $\lambda > \widetilde{\lambda}$, suppose S_1 plays its equilibrium strategy and S_2 deviates by choosing s_2 and e_2 such that it serves both buyers.[17] It must be the case that $TC\,(1)$ is violated and $TC\,(2)$ holds. S_2 cannot set s_2 and e_2 at their unconstrained maximum levels since doing so does not violate $TC\,(1)$. If it chooses s_2 and e_2 such that $TC\,(1)$ is just violated, the effort level is given by $e_2 = \frac{1}{\left(1-\gamma(1-b_1-\widehat{s}_2)^2\right)}$, where $\widehat{s}_2$ represents the constrained optimal location choice. S_2 makes

$$\frac{\left(1-\gamma\left(b_2-\widehat{s}_2\right)^2\right)}{2\left(1-\gamma\left(1-b_1-\widehat{s}_2\right)^2\right)}+\frac{1}{2}-\max\left\{0,\frac{\left(1-\gamma\lambda^2\right)}{8}\right\}-\frac{1}{2\left(1-\gamma\left(1-b_1-\widehat{s}_2\right)^2\right)^2}. \tag{A.19}$$

After redefining S_2's choice variable as $\sigma = 1 - b_1 - s_2$ and letting $\widehat{\sigma} = 1 - b_1 - \widehat{s}_2$, the net benefit from deviating is given by

$$\Delta = \frac{1-\gamma\left(\widehat{\sigma}-\lambda\right)^2}{2\left(1-\gamma\widehat{\sigma}^2\right)}+\frac{3}{8}-\frac{1}{2\left(1-\gamma\widehat{\sigma}^2\right)^2}. \tag{A.20}$$

By the envelope theorem, $\frac{d\Delta}{d\lambda} = 2\gamma\left(\widehat{\sigma}-\lambda\right) < 0$ since $\widehat{\sigma} < \lambda$. If $\lambda = 0$, $\Delta > 0$ since $\widehat{\sigma} = 0$. If $\lambda = \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$, it is straightforward to verify that $\Delta < 0$ by substituting for $\gamma = \frac{4-2\sqrt{2}}{\lambda^2}$ in (A.20) and evaluating the resulting expression graphically for $\frac{\widehat{\sigma}}{\lambda} \in [0,1]$. Hence, there must exist a critical $\lambda = \widetilde{\lambda} \in \left(0, \widehat{\lambda}\right)$ such that for $\lambda > \widetilde{\lambda}$, the candidate equilibrium is an equilibrium and for $\lambda < \widetilde{\lambda}$, it is not. ■

Claim 3 *If $\lambda > \widehat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$, S_i trades with B_j for $i,j = 1,2$ and $i \neq j$ in the second stage of the bargaining game. The optimal choices are $s_i = 1 - b_j$ and $e_i = 1/2$.*

Proof. Each firm makes $\frac{1}{8}$ in equilibrium. Both $TC\,(1)$ and $TC\,(2)$ have to be violated for this equilibrium to exist. This requires $\lambda > \frac{1}{\sqrt{2\gamma}}$. To see that there are no profitable deviations for $\lambda > \widehat{\lambda}$, suppose S_1 plays its equilibrium strategy and S_2 deviates by choosing s_2 and e_2 such that it serves both buyers.[18] It sets $s_2 = b_2 + \frac{\lambda}{2}$ and $e_2 = 1 - \frac{\gamma\lambda^2}{4}$. Note that $TC\,(1)$ is always violated and $TC\,(2)$ is satisfied for $\lambda > \sqrt{\frac{2}{\gamma}}$ at these location and investment

[17] Note that as long as $\left(1-\gamma\lambda^2\right) > 0$, deviating and serving B_1 only is not a feasible option since B_2 would continue to buy from S_2. On the other hand, if $\left(1-\gamma\lambda^2\right) < 0$, S_2 cannot earn a higher payoff by deviating and trying to serve B_1 given the high amount of investment it would have to make to have $TC\,(1)$ violated.

[18] Note that deviating and serving B_2 only cannot be a profitable deviation since S_2 can capture a greater share of the surplus created if it trades with B_1 in the second stage of the bargaining game. This is because S_2 would be negotiating with B_2 in the first stage of the bargaining game when B_2 has the option of negotiating with S_1 if the negotiation with S_2 fails. In contrast, if the negotiation between S_2 and B_1 fails, B_1 does not have access to an alternative supply of inputs.

choices. S_2's payoff if it sells to both buyers is $\frac{\left(4-\gamma\lambda^2\right)^2}{32}-\frac{1}{8}$. This implies that the net benefit from deviating is ≥ 0 for $\lambda<\widehat{\lambda}=\sqrt{\frac{4-2\sqrt{2}}{\gamma}}$. ∎

The three claims jointly imply the results stated in Proposition 3.

5 Proof of Corollary 1

It is shown in the proof of Proposition 3 that $\widehat{\lambda}(\gamma)=\sqrt{\frac{4-2\sqrt{2}}{\gamma}}$. This is clearly decreasing in γ. To see that $\widetilde{\lambda}(\gamma)$ is also decreasing in γ, note that it is defined by $\Delta=0$, where Δ is given by (A.20). Hence, by the implicit function theorem,

$$\frac{\partial\widetilde{\lambda}}{\partial\gamma}=-\frac{\partial\Delta/\partial\gamma}{\partial\Delta/\partial\lambda}.$$

In the denominator, $\frac{\partial\Delta}{\partial\lambda}=2\gamma\left(\widehat{\sigma}-\lambda\right)<0$ since $\widehat{\sigma}<\lambda$. The expression in the numerator is also negative since

$$\frac{\partial\Delta}{\partial\gamma}=\frac{\widehat{\sigma}^2\left[\left(1-\gamma\left(\widehat{\sigma}-\lambda\right)^2\right)\left(1-\gamma\widehat{\sigma}^2\right)-2\right]-\left(1-\gamma\widehat{\sigma}^2\right)^2\left(\widehat{\sigma}-\lambda\right)^2}{2\left(1-\gamma\widehat{\sigma}^2\right)^3}<0.$$

Hence, $\widetilde{\lambda}(\gamma)$ is decreasing in γ.

6 Proof of Lemma 2

Consider the SPNE outcome where B_i buys inputs from S_i for $i=1,2$. This implies $s_i=b_i$. If B_i were to use S_j's input instead, the benefit would be $e_j\left(1-\gamma\lambda^2\right)$. This expression is negative for $\lambda>\frac{1}{\sqrt{\gamma}}$. Proposition 3 states that there can be two suppliers in equilibrium for $\lambda>\widetilde{\lambda}$. Hence, to prove the result stated in the lemma, it is necessary to show that $\widetilde{\lambda}>\frac{1}{\sqrt{\gamma}}$.

Suppose not. Then neither supplier should have a profitable deviation at $\lambda=\frac{1}{\sqrt{\gamma}}$. Suppose, as in the proof of Proposition 3, S_1 plays its equilibrium strategy and S_2 deviates by choosing s_2 and e_2 such that it serves both buyers. S_2's net benefit from deviating is given by (A.20), where $\widehat{\sigma}=1-b_1-\widehat{s}_2$ stands for the distance between S_2's optimal location and B_1. Note that since S_2 deviates from its equilibrium strategy to serve both buyers, its optimal location must be between the two buyers. Hence, after substituting for $\gamma=\frac{1}{\lambda^2}$, it is straightforward to see that for a given λ value, the supplier chooses a value of $\frac{\widehat{\sigma}}{\lambda}\in[0,1]$. Since it is sufficient to find only one profitable deviation, consider $\frac{\sigma}{\lambda}=0.4$. The net benefit from deviating is positive at $\frac{\sigma}{\lambda}=0.4$ and, hence, it must be the case that $\widetilde{\lambda}>\frac{1}{\sqrt{\gamma}}$.

7 Proof of Proposition 4

(i) Suppose $\widetilde{\lambda}(\gamma) > 1$. Proposition 3 states that for all $\lambda < \widetilde{\lambda}(\gamma)$, only one supplier invests a positive amount and locates between the two buyers. Proposition 2 implies that in such cases the buyers may choose intermediate locations for sufficiently small values of t.

(ii) Suppose $\widetilde{\lambda}(\gamma) < 1$. I show that there exist values of t and γ such that the following strategies constitute an equilibrium. In the first stage of the game, buyer B_i chooses $b_i = 0$ for $i = 1, 2$. In the second stage, supplier S_i chooses $s_i = b_i$ and $e_i = \frac{1}{2}$ for $i = 1, 2$.

Consider γ sufficiently high such that $\widehat{\lambda}(\gamma) = \sqrt{\frac{4-2\sqrt{2}}{\gamma}} < 1$. By Proposition 3, for all λ values such that $\widehat{\lambda} < \lambda \leq 1$, there are two suppliers making positive investments in equilibrium. Does either of the buyers have an incentive to unilaterally deviate? Suppose B_i chooses $b_i > 0$ such that in the second stage of the game, both suppliers still make positive investments and produce specialized inputs. Lemma 2 implies that B_i's payoff is given by

$$\pi_i = t\frac{\lambda(3 + b_i - b_j)^2}{18} - K + \frac{1}{8}, \tag{A.21}$$

which is strictly decreasing in b_i. Hence, this cannot be a profitable deviation.

Now suppose B_i chooses $b_i > 0$ such that in the second stage of the game, only one supplier invests a positive amount and produces a non-specialized input. I next show that such a deviation would not be profitable for sufficiently high values of t. To see this, first note that Proposition 2 implies that for sufficiently high values of t, B_i's unconstrained location choice is $b_i = 0$ even if there will be only one non-specialized supplier in the market. Hence, by choosing $b_i > 0$, B_i must be achieving a lower payoff than it would achieve at $b_i = 0$. Second, the following lemma implies that for $\widehat{\lambda} < 1$, B_i's payoff at $\lambda = 1$ (i.e., when $b_i = b_j = 0$) is higher if there are two specialized suppliers in the market than if there is a single non-specialized supplier.

Lemma 3 *For a given value of λ, let π_i^1 and π_i^2 stand for B_i's payoff when the buyers are served by a single non-specialized supplier and two fully specialized suppliers, respectively. $\pi_i^1 \lesseqqgtr \pi_i^2$ for $\lambda \gtreqqless \widehat{\lambda}$.*

Proof. It is shown in Proposition 3 that if there is a single non-specialized supplier in the market, the value of the input to both buyers is positive if $\lambda < \frac{2}{\sqrt{\gamma}}$. Suppose this condition holds, which implies that

$$\pi_i^1 = t\frac{\lambda(3 + b_i - b_j)^2}{18} - K + \frac{1}{4}\left(1 - \gamma\frac{\lambda^2}{4}\right)^2. \tag{A.22}$$

Lemma 2 implies that if there are two specialized suppliers in the market, B_i's payoff is given by (A.21). Subtracting π_i^1 from π_i^2 yields $\frac{1}{8} - \frac{1}{4}\left(1 - \gamma\frac{\lambda^2}{4}\right)^2$, which is < 0 for $\lambda = 0$, $= 0$ for $\lambda = \widehat{\lambda}$, and is increasing in λ for all $\lambda < \frac{2}{\sqrt{\gamma}}$. Hence, it is negative for all $\lambda < \widehat{\lambda}$ and positive for all $\widehat{\lambda} < \lambda < \frac{2}{\sqrt{\gamma}}$. For $\lambda > \frac{2}{\sqrt{\gamma}}$, it is trivially the case that $\pi_i^2 > \pi_i^1$ since

$$\pi_i^1 = t\frac{\lambda\,(3 + b_i - b_j)^2}{18} - K. \tag{A.23}$$

■

Hence, choosing $b_i > 0$ such that the buyers are served by a single non-specialized supplier must yield a strictly lower payoff than the equilibrium payoff level. I conclude that for sufficiently high values of t and γ, there are no incentives to unilaterally deviate from the equilibrium strategies stated above.

(iii) Finally, I show that there is no SPNE where there are two fully specialized suppliers in the market and the buyers choose intermediate locations. Suppose not. That is, suppose that there is a SPNE where there are two fully specialized suppliers in the market and the buyers choose intermediate locations (i.e., $b_i > 0$ for $i = 1, 2$). From Proposition 3, it must be the case that $\lambda \geqslant \widetilde{\lambda}$ or else one of the suppliers would have incentives to deviate. Lemma 2 implies that B_i's payoff is given by (A.21).

To see whether there are any profitable deviations, suppose B_i marginally decreases b_i. Depending on the magnitude of λ at this candidate equilibrium and on the suppliers' strategies, if B_i makes such a deviation, the buyers may either be served by a single non-specialized supplier or they may continue to be served by two fully specialized suppliers. Suppose the buyers continue to be served by two fully specialized suppliers. Since (A.21) is decreasing in b_i, such a deviation must strictly increase B_i's payoff. Now suppose that marginally decreasing b_i leads to the buyers being served by a single supplier. Note that a necessary condition for this outcome is $\widetilde{\lambda} \leqslant \lambda < \widehat{\lambda}$. By Lemma 3, this deviation must also strictly increase B_i's payoff.

Hence, regardless of the suppliers' strategies, there is always a profitable deviation from the candidate equilibrium where there are two fully specialized suppliers in the market and the buyers choose intermediate locations.