Buyer-Supplier Interaction, Asset Specificity, and Product Choice\textsuperscript{1}

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*First version*: July 2002
*This version*: May 2007

\textsuperscript{1}I am grateful to Roman Inderst (the Editor), Debby Minehart, Daniel Piccinin, Anne van den Nouweland, and two anonymous referees for their detailed comments on earlier versions of this paper. I also wish to thank Suren Basov, John Creedy, Robert Dixon, Catherine de Fontenay, Joshua Gans, Stephen King, Simon Loertscher, and seminar participants at the Australian Economic Theory Workshop (2003), the 30th Annual Conference of EARIE (2003), International Industrial Organization Conference (2004), Australian National University, University of Arkansas, and University of Oregon for their comments. Mark Chicu and Christian Roessler have provided excellent research assistance. All errors are my own.

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Abstract

Firms often differentiate their products through the use of inputs that are differentiated or specialized. The goal of this paper is to explore how the demand for specific investments may affect product variety in a bilateral duopolistic industry. I develop a model where the degree of specificity of investments is endogenously determined through the product choices of both downstream buyers and upstream suppliers of inputs. The existence of non-contractible investments results in the emergence of a single supplier of a non-specialized input. However, as the importance of having specific inputs increases, fully specialized suppliers may emerge. The buyers choose to increase their own competition by producing more similar products only in cases when the inputs are non-specialized. They do so to increase the investment incentives of the suppliers.

JEL Classification: L11, L23, R12

Keywords: asset specificity, vertical interactions, product differentiation
1 Introduction

Product choice is one of the most fundamental decisions firms have to make. Standard economic reasoning suggests that firms would want to produce differentiated products in order to reduce the competition between them.\(^1\) This reasoning rests on the assumption that once a firm makes its product choice, it can easily find the inputs it needs to produce that product variety. In reality, it is often the case that the production of different varieties requires different inputs and firms are often restricted in their product choice by the availability of inputs.\(^2\) Hence, the product choices of downstream firms cannot be analyzed in isolation from the product choices of upstream firms. Suppliers may have to make product-specific investments to develop the required inputs. In an environment of incomplete contracts, depending on the specificity of the suppliers’ investments, this suggests that there may be a potential hold-up problem.

The goal of this paper is to explore how the demand for specific investments may affect product variety in a bilateral duopolistic industry. I consider a model of differentiated products where specific investments arise because there is an ideal input type corresponding to each possible variety of a final good. The downstream firms compete against each other in the familiar Hotelling model. They can reduce their fixed costs of production by procuring inputs from the suppliers in the upstream market. The suppliers choose what type of input to produce and how much to invest in its development after the downstream firms choose which final goods to produce. Due to contractual incompleteness, the suppliers face a potential hold-up problem. The input has no value if the investment is zero. I assume that if the downstream buyer uses an input type that is different from its ideal input type, the value created is decreasing in the distance between the product choices of the buyer and the supplier. After the suppliers make their decisions, the input prices are determined by bilateral negotiations, which is a common occurrence in markets for intermediate goods. In the bargaining process, the buyers’ payoff depends on whether they have access to alternative suppliers they can trade with and the suppliers’ payoff depends on whether they have alternative uses for their inputs.

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\(^1\)Since the seminal work of Hotelling (1929), a lot of attention has been given to the question of product choice. See Thisse and Norman (1995) for a survey of the literature.

\(^2\)It is emphasized in the product management literature that the distinctiveness of two final goods may significantly depend on the distinctiveness of the inputs used in their production (Robertson and Ulrich, 1998). Krishnan and Ulrich (2001) explain the close links that exist between firms’ choices of inputs and their decisions regarding which product variety to produce.
In this context, the analysis focuses on two questions. First, when do suppliers choose to produce specialized inputs? Second, when do buyers choose to be less than maximally differentiated? The model allows us to analyze the feedback mechanism between the buyers’ and the suppliers’ decisions. The degree of product differentiation is endogenously determined in both the downstream and upstream markets. While making their product choices, the downstream firms take into account the impact of their decisions on the upstream firms. Since the suppliers make their decisions after the buyers, their decisions reflect how specialized they would like to be. Both the buyers’ and the suppliers’ product choices jointly determine the degree of asset specificity in the market. The degree of specificity of an input to a given buyer in the downstream market depends on how suitable the input is for the purposes of that buyer as well as how suitable it is for the purposes of alternative buyers.

The analysis reveals several interesting findings. I start by considering the case where the buyers are flexible in using inputs that are not perfectly suited for their purposes (i.e., the value loss from not having fully specialized inputs is relatively low). In this case, only one supplier serves the downstream market in equilibrium. With only a single supplier in the market, the buyers do not have access to specialized inputs, but the supplier invests more in the development of the input. Thus, imperfect contracting creates a trade-off between the supply of fully specialized inputs and the supply of highly developed inputs. The buyers may choose to be less than maximally differentiated because when served by a single supplier, they have incentives to move closer to that supplier to make its inputs more suitable for their purposes and to increase the supplier’s investment incentives. Hence, in contrast with the maximum differentiation result of d’Aspremont et al. (1979), firms may not always choose to be maximally differentiated in an effort to decrease the competition between themselves.

These results suggest one explanation for the use of common inputs, i.e., platform sharing, which has become a common practice in the automobile industry (Robertson and Ulrich, 1998). According to my model, firms may choose to produce similar products and, thus, use similar inputs in order to improve their suppliers’ investment incentives.

I next consider how an increase in the disutility that buyers get from using an input type that is different from their ideal input type affects the results. The analysis reveals that as the importance of having specific inputs increases, fully specialized suppliers may emerge. This is because as it becomes more costly for the buyers to use an input variety that is different from their ideal variety, they prefer to locate such that there will be two suppliers in the market. This always results in maximum differentiation.
Thus, the flexibility of buyers in using the inputs of different suppliers may affect the structure of vertically-integrated industries. Bonaccorsi and Giuri (2001) explain how the ability to benefit from economies of scale and scope have resulted in distinct network structures in the turbojet and turboprop industries. They show that in the turboprop industry, the network structure is more fragmented because of a lower ability to adapt different engine programs to different aircraft programs.3

This paper contributes to both the literature on product differentiation and the literature on the hold-up problem by bringing them together in a single model and showing the interdependence between them. Although there is an extensive literature in industrial organization on product differentiation, the focus has almost entirely been on price and product competition in the downstream market.4 Little attention has been paid to the links between the downstream and upstream markets. Three exceptions are Pepall and Norman (2001), Belleflamme and Toulemonde (2003), and Ghosh and Morita (2006). Pepall and Norman (2001) consider a model where downstream firms can produce differentiated products by using different combinations of the differentiated inputs produced by the upstream suppliers. Their goal is to study the profitability of different vertical alliances. Belleflamme and Toulemonde (2003) illustrate that downstream and upstream firms may choose to agglomerate in an effort to decrease their costs because the upstream firms experience economies of scale. Ghosh and Morita (2006) show that manufacturers may choose to produce similar products to have access to a greater pool of suppliers and reduce their procurement costs. None of these papers considers the investment choices of the suppliers. Hence, this paper differs by considering the impact of specific investments on the product choices of both upstream and downstream firms in the context of incomplete contracts.

In the literature on the hold-up problem, my analysis of the endogenous choice of input specificity is related to Choi and Yi (2000), Church and Gandal (2000), McLaren (2000), and Grossman and Helpman (2002). A core element of these papers is that an integrated supplier chooses to produce a more specific input than an unintegrated supplier.5 Thus, whereas these

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3 Eaton and Schmitt (1994) also analyze the impact of flexible manufacturing on market structure. However, they consider the product choices of downstream firms only.

4 Current theories of horizontal product differentiation have been very much influenced by the “Main Street” model of Hotelling (1929). Ever since d’Aspremont et al. (1979) challenged the “Principle of Minimum Differentiation” established by Hotelling (1929), several papers have argued for minimum differentiation based on assumptions such as collusion or consumer heterogeneity. See, for example, de Palma et al. (1985) and Friedman and Thisse (1993). The incentives to agglomerate have also been analyzed in the economic geography literature. See Fujita and Thisse (1996) for a discussion.

5 Riordan and Williamson (1985) also analyze the impact of asset specificity on firm boundaries. However,
papers study how vertical integration may affect the choice of input specificity, I study how the links between downstream and upstream firms may affect the choice of input specificity. The downstream firms’ locations are taken as given in these four papers. In contrast, I emphasize how the downstream firms may strategically commit to competition in order to alleviate lock-in on the part of their suppliers. In this respect, the paper is also related to the literature where firms may undertake strategies in order to commit themselves to leave their buyers or suppliers with higher rents in the future. For example, Williamson (1983) explains how buyers may use hostages to support exchanges and encourage investment in specific technologies. Farrell and Gallini (1988) and Shepard (1987) analyze the strategic use of commitment to competition in order to alleviate consumer lock-in.6

The paper proceeds as follows. The details of the model are presented in the next section. Section 3 considers the case where the buyers are flexible in using alternative inputs. Section 4 extends the analysis in Section 3 by considering the impact of an increase in the importance of having specialized inputs. Section 5 concludes by suggesting avenues for future research. All proofs are in the Appendix.

2 Model

To capture the impact of specific investments on product variety in a bilateral duopoly, I extend the standard Hotelling model of spatial competition by incorporating upstream suppliers of inputs. Two downstream firms compete in a differentiated final goods market and two upstream firms sell inputs to the downstream firms. The varieties of the final good are differentiated in two respects. First, consumers regard different products as imperfect substitutes. Second, each type of final good has an ideal input type associated with it. I refer to the downstream firms as buyers (of inputs) and the upstream firms as suppliers.

The timing of the game is as follows. In the first stage, the two buyers, $B_1$ and $B_2$, choose the varieties of the final good they would like to produce. Production entails a fixed cost of production. Each buyer can use a single unit of an input developed by a supplier to decrease its fixed cost of production. In the second stage, the two upstream suppliers, $S_1$ and $S_2$, choose both which input type to produce and how much to invest in its development. Due to theend of the text.
to contractual incompleteness, the terms of trade are determined ex post through bilateral negotiations in the third stage. Finally, in the fourth stage, the buyers compete in the final goods market by choosing prices simultaneously and make sales.

To capture the consumers’ demand for differentiated products, I employ the setting of d’Aspremont et al. (1979), which modifies the standard Hotelling (1929) model of spatial competition by assuming quadratic consumer transportation costs. The market region is described by the unit interval $[0, 1]$. There is a unit mass of consumers who are uniformly distributed along this interval. Each consumer’s location, denoted by $x \in [0, 1]$, represents that consumer’s ideal variety. The two downstream buyers, $B_1$ and $B_2$, locate at distances $b_1$ and $b_2$ from the two ends of the unit line. Without loss of generality, I assume that $B_1$ is located to the left of $B_2$: $0 \leq b_1 \leq 1 - b_2 \leq 1$.

Each consumer purchases exactly one unit from the firm offering it at the lower effective price, namely the mill price plus the transportation cost. This implies that a consumer of type $x$ purchasing from $B_1$ and $B_2$ gets utility $U(x, b_1, p_1) = S - p_1 - t(x - b_1)^2$ and $U(x, b_2, p_2) = S - p_2 - t(x - (1 - b_2))^2$ respectively, where $S$ represents the gross surplus from consumption, $p_1$ and $p_2$ stand for the prices at which the buyers offer their goods, and $t$ is a measure of consumer loyalty. I assume $S$ is sufficiently high so that all consumers buy. Disutility costs vary quadratically with the distance between the product produced by the firm and that which is most preferred by the consumer. As shown in d’Aspremont et al. (1979), this assumption prevents discontinuities in the profit functions of the firms.

Production of the final goods entails a fixed cost and entry into the market implies a commitment to incur the fixed cost of production. This assumption implies that the buyers make the production decision at the same time as the location decision since they cannot avoid the fixed cost by not producing later in the game. To illustrate, the fixed cost of production may be the cost of abiding by some government regulation. For example, in the mining industry, firms in most jurisdictions are subject to clean-up costs, which are the costs of securing, decommissioning, and dismantling of mining facilities. Firms commit to incur these costs when they open a mining facility in a specific location. However, they can obtain specialized technologies or machineries which may reduce the contamination during the production process or the costs of cleaning up after the production process.

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7I am grateful to Daniel Piccinin for suggesting this example. The mine clean-up procedure can involve substantial costs and can take years to complete. See, for example, the Columbus Dispatch (2006).

8Alternatively, one can interpret the current set-up as a situation where the buyers are committed to production due to contracts they have signed prior to entering the market. For example, the buyers may
Before entry, the profit functions of the buyers have the general form

$$\pi_i (p_1, p_2) = (p_i - c) D_i (p_1, p_2) - F_i, \quad i = 1, 2$$

(1)

where \( D_i (p_1, p_2) \) stands for the demand for buyer \( B_i \)'s product, and \( c \) and \( F_i \) denote the marginal and fixed cost of production respectively. As mentioned above, the buyers can reduce the fixed cost of production with the purchase of a single unit of a customized, location-specific input.\(^9\) If \( B_i \) does not purchase any inputs, \( F_i = K \). Thus, I assume that the firms are initially symmetric in their costs of production. More generally, the specific value \( F_i \) takes is revealed after the input purchases and prices are determined by bilateral negotiations in the input market as described below. In the following analysis, I assume \( F_i \) is sufficiently low such that both buyers find it profitable to enter the market.

To obtain the inputs they need, the buyers contact the suppliers in the upstream market after making their location decisions. The location of the final good in the market interval represents the specification of the input needed in its production. Hence, the unit interval represents both the space of possible input types and the space of possible final goods. I assume that \( B_i \) contacts \( S_i \), where \( i = 1, 2 \), and provides the supplier with specific information on the characteristics of the input it requires. The information provided by the buyers allows the suppliers to locate anywhere they choose in the input space.

The benefit each buyer receives from the input it uses depends on how close the input is to the buyer’s ideal variety as well as how much effort the supplier exerts in its development. I assume that due to the uncertain nature of the innovation process, the parties cannot sign enforceable ex ante contracts for the delivery of a specific input (Aghion and Tirole, 1994). The precise nature of the required input is revealed only ex post and the parties cannot commit to refrain from renegotiation.\(^10\) As is well known, the absence of ex ante contracts results in the hold-up problem in the investment stage. Since my goal is to analyze the impact of the hold-up problem on the firms’ product choices, I assume that vertical integration is not a viable way of dealing with the potential underinvestment problem.

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\(^9\)Assuming that the inputs decrease the buyers' fixed cost of production, rather than possibly both their marginal and fixed costs of production, makes the analysis more tractable. I briefly discuss in Section 5 how changing this assumption would affect the analysis.

\(^10\)For example, Helper and Levine (1992) state that in the automotive industry, contracts with suppliers are necessarily incomplete because of the complexity of the parts. They state that ‘since engineering changes are common, the part actually produced by the supplier is often not the same as the part that was contracted for’ (p. 567).
In the absence of ex ante contracts, the suppliers choose both the input variety they would like to produce and their effort level. They locate at distances $s_1$ and $s_2$ from the two ends of the unit line. A buyer located at $b_i$ gets the maximum benefit from using the input produced by the supplier located at $s_i = b_i$. $B_i$ can also use an input produced by a supplier located at a different point along the unit interval, but the benefit from using a less-than-ideal input decreases with the degree of differentiation between the locations of the buyer and the supplier. Specifically, since I would like to capture the interaction between the location and investment choices of the suppliers, I assume that the input $B_i$ procures from $S_i$ reduces its fixed cost of production by

$$e_i \left(1 - \gamma (b_i - s_i)^2\right),$$

(2)

where $\gamma$ represents the importance of specificity in the input benefit function.\textsuperscript{11} A lower value of $\gamma$ corresponds to a higher flexibility in using input varieties that are different from the buyer’s ideal variety.\textsuperscript{12} This formulation assumes that the marginal benefit of effort depends on the distance between $S_i$ and $B_i$, $|b_i - s_i|$. Hence, it implies that the suppliers’ investment decisions are affected by their and the buyers’ location choices. If $b_i = s_i$, $B_i$ receives a fixed cost reduction of $e_i$. When $b_i \neq s_i$, the marginal disutility from using an input variety that is different from the ideal one is increasing in $|b_i - s_i|$. I assume the parameter values are such that the cost reduction that can be obtained from the inputs is less than or equal to $K$.

For the suppliers, the cost of exerting an effort level of $e_i$ is $e_i^2$. In the input development stage, each supplier first develops a prototype. Once the suppliers develop the prototypes, they bring them to their potential customers and the partners negotiate the terms of trade. The cost of producing additional units of the input is zero.

For the determination of the input prices, I consider a bargaining game where the existence of alternative trading partners can potentially make a difference. The bargaining game consists of two stages.\textsuperscript{13} In the first stage, $B_i$ approaches the supplier it has contacted after choosing its location and they bargain over price. If the negotiations between $B_i$ and $S_i$ fail, they can try to find other partners in the second stage of bargaining and bargain with them.

\textsuperscript{11}If $B_i$ purchases the input from $S_j$ instead, its fixed cost of production is reduced by $e_j \left(1 - \gamma (1 - s_j - b_i)^2\right)$.

\textsuperscript{12}In the following analysis, I first consider the case when $\gamma = 1$. I then analyze how the results change with changes in $\gamma$ in Section 4.

\textsuperscript{13}Grossman and Helpman (2002) consider a similar two-stage bargaining model, where they assume that in the first stage, the negotiations between the parties can fail with some exogenous probability. See Inderst and Wey (2003), and de Fontenay and Gans (2005) for two more fully specified bargaining procedures which give rise to the Shapley value.
Specifically, $S_i$ can try to approach $B_j$ and $B_i$ can try to approach $S_j$, where $i \neq j$. I assume that once the first-stage negotiations fail, $B_i$ and $S_i$ cannot meet to bargain again. If the second round of negotiations also fails, there are no more opportunities for trade.

The parties split the surplus from bargaining equally in both rounds of negotiations. In the first stage of the bargaining game, the parties negotiate knowing that if their negotiations fail, they may have the option of dealing with an alternative partner in the second stage. Therefore, the second stage of the bargaining game determines what the parties would get if the negotiations failed in the first stage of the bargaining game. An increase in the payoffs in the second stage of the bargaining game improves the bargaining positions of the parties in the first stage.

I now proceed to derive the pure-strategy symmetric subgame perfect Nash equilibria (SPNE) of the entire game by using backward induction.

3 Analysis
3.1 Bargaining Outcomes

I start the analysis by considering the case when $\gamma = 1$, which implies that for the buyers, the benefit from using any input variety in the unit interval is positive as long as the supplier of that input variety invests a positive amount. As $\gamma$ increases, the buyers find it less and less appealing to use the inputs varieties that are different from their ideal variety. This is analyzed in Section 4.

To find the SPNE of the game specified in Section 2, I work backward from the determination of the output prices. Consider the final-stage subgame, where the buyers compete by choosing prices for given locations $b_1$ and $1 - b_2$. $B_i$ maximizes $(p_i - c) D_i(p_1, p_2)$. The firms’ demand functions can be determined by defining the indifferent consumer. After substituting for the demand functions in the profit functions of the firms, the Nash equilibrium in prices can be found. Solving the first-order conditions for profit maximization in prices simultaneously yields the result that buyer $B_i$’s equilibrium price is equal to

$$p^*_i(b_i, b_j) = c + \lambda \frac{(3 + b_i - b_j)}{3},$$

where $\lambda = 1 - b_i - b_j$ stands for the degree of product differentiation in the downstream market. It is straightforward to check that the second-order condition is satisfied. The equilibrium price level is not affected by the suppliers’ location and investment choices because the inputs
bought affect the fixed costs of production only.

In the third stage of the game, the parties decide whether to trade after observing the location and investment choices made in the previous stages. I solve for the SPNE of the bargaining game specified in Section 2. The following lemma states the necessary and sufficient condition for trade to take place between $B_i$ and $S_i$ in the first stage of the bargaining game.

**Lemma 1** Given $b_1, b_2, s_1, s_2, e_1$ and $e_2$, consider the trade condition $TC(i)$ defined as

$$e_i \left(1 - (b_i - s_i)^2\right) - \frac{1}{2} e_j \left(1 - (1 - s_j - b_i)^2\right) \geq 0,$$  \hspace{1cm} (4)

where $i, j = 1, 2$ and $i \neq j$.

(i) If $TC(i)$ holds for $i = 1, 2$, $B_i$ and $S_i$ trade one unit in the first stage of the bargaining game.

(ii) If $TC(i)$ holds but $TC(j)$ does not, $B_i$ buys one unit from $S_i$ in the first stage and $B_j$ buys one unit from $S_i$ in the second stage of the bargaining game.

(iii) If $TC(i)$ does not hold for $i = 1, 2$, $B_i$ buys one unit from $S_j$, where $i \neq j$, in the second stage of the bargaining game.

**Proof.** All proofs are in the Appendix. ■

Since the suppliers’ marginal cost of supplying additional units is zero, if the negotiations in the first stage of the bargaining game fail, the buyers can always find an alternative supplier to deal with in the second stage of the bargaining game. The trade condition $TC(i)$ given in (4) implies that $B_i$ and $S_i$ trade in the first stage of the bargaining game if the surplus created is nonnegative. The surplus created is the difference between the benefit of $S_i$‘s input to $B_i$ and what $B_i$ gets if the negotiations break down. Thus, trade takes place between $B_i$ and $S_i$ if it results in a higher value than $B_i$ can get by trading with $S_j$. Since the benefit of an input from a specific supplier depends on that supplier’s location and investment choices, (4) implies that $B_i$ and $S_i$ are more likely to trade as $e_i$ increases or as $S_i$ moves closer to $B_i$, and are less likely to trade as $e_j$ increases or as $S_j$ moves closer to $B_i$.

The trade condition $TC(i)$ also implies that the possibility that $S_i$ may trade with $B_j$ in the second stage does not affect the surplus created in the first stage. Since $S_i$ is not capacity-constrained and can produce additional units at zero marginal cost, it can sell units to $B_j$ in the second stage whether or not it trades with $B_i$ in the first stage. Hence, the potential
trade with $B_j$ in the second stage does not have the impact of improving $S_i$’s bargaining position in the first-stage negotiation with $B_i$.

Lemma 1 indicates that depending on whether $TC\ (i)$ holds for $i = 1, 2$, either the buyers deal with different suppliers or they both deal with the same supplier in equilibrium. In case (i), the buyers procure inputs from different suppliers in the first stage of the bargaining game. They receive their share of the surplus created and the payoff they would receive if the negotiations broke down. Hence, $B_i$ and $S_i$’s payoffs are

$$W_{B_i} = (p_i^* - c)D_i(p_i^*, p_j^* + \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right) + \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right)$$

$$W_{S_i} = \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right) - \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right).$$

In case (ii), both buyers procure inputs from the same supplier in different stages of the bargaining game. This increases supplier $S_i$’s payoff by $\frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right)$ and reduces supplier $S_j$’s payoff to zero. Hence, the payoffs are

$$W_{B_i} = (p_i^* - c)D_i(p_i^*, p_j^* + \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right) + \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right)$$

$$W_{B_j} = (p_j^* - c)D_j(p_i^*, p_j^* + \frac{1}{2}e_i\left(1 - (b_i - s_i)^2\right)$$

$$W_{S_i} = \frac{1}{2}e_i\left(2 - (b_i - s_i)^2 - (1 - b_j - s_i)^2\right) - \frac{1}{4}e_j\left(1 - (1 - s_j - b_i)^2\right)$$

$$W_{S_j} = 0.$$ (10)

Finally, in case (iii), the buyers procure inputs from different suppliers in the second stage of the bargaining game. They receive their share of the surplus created. Hence, $B_i$ and $S_i$’s payoffs are

$$W_{B_i} = (p_i^* - c)D_i(p_i^*, p_j^* + \frac{1}{2}e_i\left(1 - (1 - s_j - b_i)^2\right)$$

$$W_{S_i} = \frac{1}{2}e_i\left(1 - (1 - b_j - s_i)^2\right).$$ (12)

A comparison of (11) and (5) shows that having access to alternative suppliers helps the buyers. If $TC\ (i)$ holds, $B_i$’s payoff is given by its share of the surplus created when it trades with $S_i$ and the payoff it would receive if the negotiations broke down. If $TC\ (i)$ does not hold, $B_i$’s payoff is determined by its share of the surplus created when it trades with $S_j$ since the payoff it would receive if the negotiation with $S_j$ broke down is zero. This is because trade takes place in the second stage of the bargaining game and, as stated in Section 2, if the second-stage negotiations fail, there are no more opportunities for trade.
3.2 Product and Investment Choices in the Upstream Market

In the second stage of the game outlined in Section 2, the suppliers simultaneously choose the input variety they would like to produce and their investment levels. $S_i$ maximizes its earnings net of investment costs taking $b_i, b_j, s_i, s_j$ and $e_j$ as given. Its earnings in the input market for any given set of $b_i, b_j, s_i, s_j, e_i$ and $e_j$ depend on whether $TC(i)$ holds for $i = 1, 2$ and are as presented in Section 3.1.

Since the suppliers choose their locations after the buyers do, their decisions reflect the extent to which their production is customized to a specific buyer. Proposition 1 states that there is only one supplier choosing a positive amount of investment in equilibrium. The second supplier does not find it profitable to enter the market by investing a positive amount.

**Proposition 1** In equilibrium, both buyers purchase inputs from the same supplier, which chooses to produce an input type that is exactly in the middle between the two buyers’ ideal input types. The supplier’s investment level increases as the degree of product differentiation in the downstream market decreases.

In the proof of Proposition 1, it is shown that even in cases when the second supplier could enter and produce a fully specialized input type, the buyers prefer to use a common input type that is not fully specialized for their purposes in order to benefit from the increased investment incentives of the single supplier. When both buyers purchase inputs from the same supplier, that supplier has increased incentives to invest because of the higher demand it faces.\(^{14}\)

Suppose it is $S_i$ that serves both buyers. Its payoff is given by (9). For given values of $b_i$ and $b_j$, $S_i$’s equilibrium choices of location and investment are

$$s_i^*(b_i, b_j) = b_i + \frac{\lambda}{2} \quad (13)$$

and

$$e_i^*(b_i, b_j) = 1 - \frac{\lambda^2}{4}. \quad (14)$$

The investment level increases as the buyers locate closer to each other because, given the supplier’s optimal location choice, a decrease in the distance between the two buyers implies a

\(^{14}\)To gain an understanding of the role investment plays in the model, it is instructive to consider briefly what the suppliers’ location choices would be if they were not making any investment decisions. It is straightforward to conclude based on their payoff functions that if the suppliers chose locations only, they would prefer to produce fully specialized inputs. Hence, it is the interaction between the location and investment decisions of the suppliers that results in the emergence of a single supplier of a non-specialized input in the market.
decrease in the distance between the supplier and each of the buyers. This causes the buyers to receive a higher benefit from the inputs they obtain and the supplier to achieve a higher payoff.

3.3 Product Choices in the Downstream Market

In the first stage of the game, the buyers choose their locations to maximize their payoffs. This section examines how vertical interactions affect the product choices of buyers. Equation (1) implies that both buyers enter the market if \((p^*_i - c) D_i (p^*_1, p^*_2) \geq F_i\), where \(F_i\) is the net fixed cost of production after taking the cost reduction and input payment into account. In the following analysis, I restrict attention to the case when both buyers enter the market.\(^{15}\)

The results in Section 3.2 indicate that for all \(b_i, b_j \in [0, 1]\), there is only one supplier making a positive amount of investment in equilibrium. Hence, \(B_i\) and \(B_j\) maximize \(W_{B_i} - K\) and \(W_{B_j} - K\) respectively, where \(W_{B_i}\) and \(W_{B_j}\) are as defined in (7) and (8). Assuming \(S_i\) is the active supplier, \(e_j = 0\) in equilibrium. This implies that the two payoff functions are symmetric.

Substituting for \(s^*_i (b_i, b_j)\) and \(e^*_i (b_i, b_j)\) in (7) and subtracting \(K\) gives

\[
\pi_i = (p^*_i - c) D_i (p^*_1, p^*_2) - K + \frac{1}{2} e^*_i (b_i, b_j) \left(1 - (b_i - s^*_i (b_i, b_i))^2\right).
\]

This expression illustrates the several effects that shape the location choices of the buyers. The first two terms, \((p^*_i - c) D_i (p^*_1, p^*_2) - K\), represent the profit level of \(B_i\) if it does not purchase any inputs. It is easy to verify that this is decreasing in \(b_i\). In order to relax the price competition between themselves, the firms would prefer to locate as far from each other as possible. I refer to this as the *competition effect*.

The third term, \(\frac{1}{2} e^*_i (b_i, b_j) \left(1 - (b_i - s^*_i (b_i, b_j))^2\right)\), represents the benefit \(B_i\) gets from using \(S_i\)’s input. This benefit has two components. First, it depends on the level of investment made by \(S_i\). Equation (14) implies that \(e^*_i (b_i, b_j)\) is increasing in \(b_i\). An increase in \(b_i\) results in a decrease in the degree of product differentiation in the downstream market. The supplier invests more when the buyers are located closer together because the value created from the use of the inputs is higher. Hence, in order to increase the investment incentives of their suppliers, the buyers prefer to locate closer to each other. This is the *investment effect*.

\(^{15}\)After solving the buyers’ maximization problem, I specify the parameter values for which this assumption will hold in the proof of Proposition 2.
The benefit \( B_i \) gets from using \( S_i \)'s input also depends on how suitable \( S_i \)'s input is for \( B_i \)'s purposes, which is represented by \( 1 - (b_i - s_i^*(b_i, b_j))^2 \). It is easy to verify that this term is increasing in \( b_i \). This specificity effect reflects the fact that the buyers always like to locate close to the suppliers they trade with. They know that for given values of \( b_i \) and \( b_j \), the suppliers always choose locations between them. The specificity effect indicates that for a given value of \( b_j \), as \( b_i \) increases, the distance between \( B_i \) and \( S_i \) decreases and the buyer benefits from using an input which is more suitable for its purposes.

The location choice of \( B_i \) depends on which of the three effects dominate. After substituting the exact expressions for \( s_i^*(b_i, b_j) \) and \( e_i^*(b_i, b_j) \) in (15), \( B_i \) solves

\[
\max_{b_i} \pi_i = t \frac{\lambda(3 + b_i - b_j)^2}{18} + \frac{(4 - \lambda^2)^2}{32} - K
\]

taking \( b_j \) as given. Given the symmetry between the firms, I focus on symmetric equilibrium. Note that when \( t = 0 \), the optimal product choices are \( b_i^* = b_j^* = b^* = \frac{1}{7} \). That is, when the consumers are indifferent about which variety they consume, the firms prefer to locate in the middle. In the proof of Proposition 2, it is shown that as \( t \) increases, \( b^* \) monotonically decreases. Hence, the following result can be stated.

**Proposition 2** The buyers’ symmetric equilibrium location choice is decreasing in \( t \). For sufficiently low values of \( t \), the investment and specificity effects dominate the competition effect, and the buyers choose to be less than maximally differentiated.

A low \( t \) value implies that even with maximum differentiation, the buyers face intense competition because the consumers care less about consuming a variety that is different from their ideal one. Proposition 2 states that as the measure of consumer loyalty, \( t \), decreases, the buyers have increased incentives to agglomerate in the middle. As \( t \) decreases and the buyers cannot effectively decrease the rivalry between them by locating further away from each other, the investment and specificity effects start to play relatively more important roles in their location decisions. Hence, they may choose less than maximum differentiation both in order to increase the investment incentives of the suppliers and to use more suitable inputs.

The analysis reveals that once the classic Hotelling model with quadratic costs is extended to take into account the dynamics of the interactions between upstream suppliers and downstream buyers, buyers may choose intermediate locations in the product space in order to be close to the only supplier in the market which is located in the middle.
4 Changes in Relative Importance of Specificity

In the analysis so far, $\gamma$ was restricted to 1. A question that arises from the analysis in Section 3 is how the magnitude of $\gamma$ affects the equilibrium outcome. In this section, I explore whether there can be a SPNE of the entire game where both suppliers make positive investment choices.

It is straightforward to show that the trade condition given in (4) is modified in the following way.

$$e_i \left(1 - \gamma (b_i - s_i)^2\right) - \max \left\{0, \frac{1}{2}e_j \left(1 - \gamma (1 - s_j - b_i)^2\right)\right\} \geq 0. \quad (17)$$

This is because while negotiating with $S_i$, $B_i$ may have the option to trade with $S_j$ in the second stage of the bargaining process if and only if the benefit it receives from using $S_j$’s input, $e_j \left(1 - \gamma (1 - s_j - b_i)^2\right)$, is positive.

In the second stage of the game, for a given value of $\gamma$, either one supplier or both suppliers make positive investments depending on the values of $b_i$ and $b_j$. The following proposition states the SPNE outcome in the second stage of the game for different $\lambda$ values.

**Proposition 3** In the second stage of the game, there exist two critical $\lambda$ values, $0 < \tilde{\lambda}(\gamma) < \hat{\lambda}(\gamma)$, such that:

(i) For $\lambda < \tilde{\lambda}(\gamma)$, only one supplier chooses a positive investment level and locates between the two buyers.

(ii) For $\tilde{\lambda}(\gamma) \leq \lambda < \hat{\lambda}(\gamma)$, there are two types of equilibria. In one of them, only one supplier chooses a positive investment level and locates between the two buyers. In the other one, both suppliers choose positive investment levels. $S_i$ serves $B_i$ and chooses $s_i = b_i$.

(iii) For $\lambda \geq \hat{\lambda}(\gamma)$, there are two types of equilibria, both of which have both suppliers choosing positive investment levels and producing fully specialized inputs. In one of them, $S_i$ serves $B_i$ and in the other one, $S_i$ serves $B_j$ for $i = 1, 2$ and $i \neq j$.

Proposition 3 states that for a given $\gamma$ value, if the degree of product differentiation in the downstream market is sufficiently small, there will be only one supplier choosing a positive investment level and serving both buyers. If the degree of product differentiation is sufficiently large, there exist SPNE outcomes where both suppliers make positive investment levels and choose to produce fully specialized inputs. This is because as the degree of product differentiation increases, it becomes too costly to use non-specialized inputs even if doing so
results in higher investment incentives. The proposition also states that there exists a range of \( \lambda \) values for which there is a multiplicity of equilibria in the second stage of the game.

**Corollary 1** As \( \gamma \) increases, it becomes less desirable to have one common supplier and more desirable to have two fully specialized suppliers.

In the proof of Corollary 1, I show that the critical values \( \lambda(\gamma) \) and \( \bar{\lambda}(\gamma) \) given in Proposition 3 are decreasing in \( \gamma \). As \( \gamma \) increases, the buyers get higher disutility from using an input type that is not perfectly suited to their needs. This is the reason the range of \( \lambda \) values for which only one supplier serves both buyers in equilibrium gets smaller. On the other hand, for sufficiently small values of \( \gamma \) such that \( \lambda(\gamma) > 1 \), there exists a unique SPNE where a single supplier serves both buyers. The analysis in Section 3 implies that the critical \( \gamma \) value such that \( \lambda(\gamma) = 1 \) must be greater than 1.

I next show that whenever \( \lambda \) is high enough for both suppliers to make positive investments, the benefit the buyers get from using the input of the alternative supplier is zero. That is, they do not have the option of obtaining inputs from the alternative supplier if the negotiation in the first stage fails.

**Lemma 2** For all \( \lambda \) values such that there are two suppliers making positive investments in equilibrium (i.e., for \( \tilde{\lambda}(\gamma) \leq \lambda \leq 1 \)), the buyers do not benefit from the inputs of the alternative suppliers.

In the proof of Lemma 2, it is shown that the range of \( \lambda \) values such that the buyers benefit from having access to alternative suppliers is smaller than the range of \( \lambda \) values such that a single supplier serves both buyers. This implies that whenever the buyers receive a positive benefit from using the input of the alternative supplier located at distance \( \lambda \) from them (i.e., where the rival buyer is located), one of the suppliers has an incentive to deviate and locate such that it serves both buyers. This is because if a supplier deviates to serve both buyers, it locates between the two buyers. Hence, the distance between the supplier and each of the buyers is less than \( \lambda \), the distance between the two buyers. This guarantees that both buyers receive a net benefit from using the deviating supplier’s inputs and implies that the deviation will be profitable.

An increase in the number of input varieties in the market can have two kinds of effect on the buyers’ payoff. The direct effect is that with more variety, each buyer may be able
to find an input type which is more suited to its needs. The indirect effect is that having an alternative supplier to deal with increases the buyers’ bargaining power. Corollary ?? implies that even if the number of varieties in the market increases, the buyers do not benefit from having access to an alternative supplier in the bargaining process. The existence of two varieties helps the buyers in getting more specialized inputs only. Hence, competition between the suppliers never arises in the sense that the suppliers are never harmed in the bargaining process by the existence of an alternative supplier in the market.

I now turn to the equilibrium of the entire game. The following proposition states that there exist parameter values for which both suppliers make positive investments in equilibrium.

Proposition 4 (i) If $\tilde{\lambda}(\gamma) > 1$, then the unique type of equilibrium involves one supplier serving both buyers. The supplier locates exactly in the middle between the two buyers and the buyers choose intermediate locations for sufficiently small $t$ values. (ii) If $\tilde{\lambda}(\gamma) < 1$, there exist values of $t$ and $\gamma$ such that both suppliers making positive investments and producing fully specialized inputs is a SPNE. (iii) There is no SPNE where both of the suppliers make positive investments and the buyers choose intermediate locations.

Part (i) of Proposition 4 extends the result from Section 3. For values of $\gamma$ such that $\tilde{\lambda}(\gamma) > 1$, one supplier serving both buyers is the SPNE. Part (ii) of Proposition 4 states that for $\tilde{\lambda}(\gamma) < 1$, the buyers may prefer to locate such that both suppliers make positive investments in equilibrium. Finally, part (iii) of Proposition 4 implies that one would expect to see the buyers choosing intermediate locations only in cases when both buyers are served by the same supplier. In an industry where multiple suppliers serve downstream buyers by producing specialized inputs, one would not expect to see less than maximum differentiation in the downstream market. In the subgames where there will be two suppliers in the market, the buyers choose maximum differentiation because they do not benefit from the inputs of the alternative supplier.

5 Conclusion

Firms often differentiate their products through the use of inputs that are differentiated or specialized. This implies that product choices made in the downstream markets cannot be analyzed in isolation from the product choices made in the upstream markets. The goal of
this paper has been to explore how the demand for specific investments in the production of inputs may affect product variety in a bilateral duopolistic industry. The degree of specificity is endogenously determined in the model through the product choices made by all the firms in the market. This is because if the buyers produce more similar products, the suppliers’ inputs become less specialized.

I have specifically studied when suppliers choose to produce specialized inputs and when buyers may choose to be less than maximally differentiated. The analysis shows that two things play critical roles in the decisions of the suppliers: (i) the degree of product differentiation in the downstream market, and (ii) the importance of having specialized inputs. The benefit from a specific input type depends on how well suited the input is for the needs of the buyer and how much effort the supplier has put into its development. Under incomplete contracts, there may be a trade-off between these two. This implies that if the suppliers have no capacity constraints and if the buyers are flexible in using inputs that are not perfectly suited for their purposes, only one supplier serves the downstream market in equilibrium. The supplier chooses to locate between the two buyers. With only a single supplier in the market, the buyers do not have access to specialized inputs, but the supplier invests more in the development of the input.

As the importance of having specific inputs increases, fully specialized suppliers may emerge. It becomes unprofitable for the buyers to locate such that only one supplier invests in equilibrium because this would require them to approach each other too much. Instead, they choose to be maximally differentiated. Hence, the buyers choose intermediate locations only in cases when they want to encourage entry by a single supplier.

An important way in which the current analysis can be extended is by considering the case where inputs can determine both the marginal and fixed costs of production. If the suppliers were to invest to decrease the buyers’ marginal cost of production, a change in the locations of the buyers would affect the suppliers’ payoffs through two different channels. The suppliers’ payoffs and investment incentives would change because of a change in both the profits made in the downstream market and the benefits obtained from the inputs. As in McLaren (2000), the current set-up has allowed me to differentiate between these two effects and concentrate on the latter one. With the insights from the current analysis, it would be worthwhile to analyze the case when inputs determine the marginal cost of production.
References


Appendix

This Appendix contains the proofs of Corollary 1, Lemmas 1-2, and Propositions 1-4.

1 Proof of Lemma 1

At the end of the first stage, there are 3 types of subgames: 1) Both pairs have traded 1 unit in the first stage, 2) one of the pairs has traded 1 unit and the other pair has traded 0 units in the first stage, and 3) both pairs have traded 0 units in the first stage. In a subgame of type 1, no more trade takes place in the second stage because each buyer can use at most 1 unit. In the other two types of subgames, the buyers with 0 units can obtain inputs in the second stage. Since the suppliers are not capacity-constrained, their ability to do so does not depend on whether or not trade took place between the other pair in the first stage.

Consider the negotiation between $B_i$ and $S_j$ in the second stage. The gains from trade are $e_j\left(1 - (1 - b_i - s_j)^2\right)$, which is the benefit $B_i$ gets from using one unit of the input produced by $S_j$. Hence, $B_i$ buys an input from $S_j$ in the second stage if it has 0 units at the end of the first stage and if $e_j > 0$. If trade take place, $B_i$ and $S_j$ receive the sum of their share of this amount and their outside option. $B_i$'s outside option is determined by the amount it would make in the downstream market. $S_j$'s outside option is 0 since it cannot find another trading partner after the second stage of the bargaining game is over.

In the first stage of the bargaining game, $B_i - S_i$ and $B_j - S_j$ bargain simultaneously. To determine mutual best responses, consider the negotiation between $B_i$ and $S_i$. Suppose $B_j$ and $S_j$ do not trade. $S_i$ can negotiate with $B_j$ in the second stage whether or not it trades with $B_i$ in the first stage. If $e_i\left(1 - (1 - b_j - s_i)^2\right) > 0$, it sells 1 unit to $B_j$ and earns $\frac{1}{2}e_i\left(1 - (1 - b_j - s_i)^2\right)$. If $B_i$ and $S_i$ do not trade, $B_i$ can negotiate with $S_j$. If it trades with $S_j$, the net benefit it gets is $\frac{1}{2}e_j\left(1 - (1 - s_j - b_i)^2\right)$. Hence, $B_i$ and $S_i$’s joint profits in case of not trading are

$$ (p_i^* - c) D_i (p_1^*, p_2^*) + \frac{1}{2}e_i\left(1 - (1 - s_j - b_i)^2\right) + \frac{1}{2}e_j\left(1 - (1 - b_j - s_i)^2\right). $$ (A.1)

If $B_i$ and $S_i$ trade 1 unit, their joint profits are

$$ (p_i^* - c) D_i (p_1^*, p_2^*) + e_i\left(1 - (b_i - s_i)^2\right) + \frac{1}{2}e_i\left(1 - (1 - b_j - s_i)^2\right). $$ (A.2)

Comparing the joint profits reveals that $B_i$ and $S_i$ trade 1 unit if (4) holds and trade 0 units if it does not hold.
A similar analysis reveals that the best response of $B_i$ and $S_i$ in the case when $B_j$ and $S_j$ trade unit is determined by (4) also. Hence, for every possible strategy of $B_j$ and $S_j$, $B_i$ and $S_i$ trade if (4) holds and do not trade if it does not.

Labelling the inequality in (4) as $TC(i)$, we get the following cases. If both $TC(i)$ and $TC(j)$ hold, both pairs trade in the first stage of the bargaining game. If only $TC(i)$ holds, $B_j$ and $S_j$ do not trade in the first stage of the bargaining game. If neither $TC(i)$ nor $TC(j)$ holds, none of the pairs trades in the first stage of the bargaining game. The firms’ payoffs in the different cases are given in Section 3.1.

2 Proof of Proposition 1

The proof proceeds in two steps.

(i) I first show that neither supplier has an incentive to deviate from the equilibrium stated in Proposition 1. In equilibrium, supplier $S_i$ earns the maximum possible profit by being the sole supplier of inputs to both buyers. Hence, it cannot do any better by deviating. $S_j$, on the other hand, makes 0. It is necessary to check whether it can earn a positive profit by setting $s_j$ and $e_j$ such that it trades with $B_j$ only, with $B_i$ only, or with both of the buyers.

If it trades with $B_j$, its payoff is equal to $WS_j - \frac{e_j^2}{2}$, where $WS_j$ is given by (6) after switching $i$ and $j$. The unconstrained optimum is $s_j = b_j$ and $e_j = \frac{1}{2}$. For $B_j$ to prefer to trade with $S_j$, $TC(j)$ must be holding. Given the equilibrium strategy of $S_i$, this condition is satisfied. Substituting for the relevant choices of $s_i$, $s_j$, $b_i$ and $b_j$ yields $WS_j = \frac{1}{8} - \frac{(4-\lambda^2)^2}{64}$, which is negative for all $\lambda \in [0, 1]$. Hence, this is not a profitable deviation.

If it trades with $B_i$, it makes $WS_j - \frac{e_j^2}{2}$, where $WS_j$ is given by (12) after switching $i$ and $j$. The unconstrained optimum is $s_j = 1 - b_i$ and $e_j = \frac{1}{2}$. For $B_i$ to prefer to buy from $S_j$ instead of from $S_i$, it must be the case that $TC(i)$ is violated. Given the equilibrium strategy of $S_i$, $TC(i)$ is satisfied at the unconstrained optimum stated above. Supplier $S_j$ can choose $s_j$ and $e_j$ such that it is just violated. The constrained optimal choices are $s_j = 1 - b_i$ and $e_j = \frac{(4-\lambda^2)^2}{8}$. Evaluating $S_j$’s profit level at these choices gives

$$
\left(\frac{(4-\lambda^2)^2}{16}\right)\left(1 - \frac{(4-\lambda^2)^2}{8}\right).
$$

(A.3)

Again, since this is negative for all $\lambda \in [0, 1]$, this is not a profitable deviation.

Finally, $S_j$ can set $s_j$ and $e_j$ such that both buyers prefer to trade with it. This implies that its choices must violate $TC(i)$ and satisfy $TC(j)$. $S_j$’s payoff is equal to $WS_j - \frac{e_j^2}{2}$,
where $W_{S_j}$ is given by (9) after switching $i$ and $j$. If $TC(i)$ and $TC(j)$ are evaluated at the unconstrained optimum of $S_j$’s payoff function, both of them are satisfied, which implies that $B_i$ prefers to trade with $S_i$ instead of $S_j$. Hence, consider values of $s_i$ and $e_i$ such that $TC(i)$ is just violated. Note that if there is not a profitable deviation that satisfies only one of the constraints, there cannot be a profitable deviation that satisfies both constraints. Setting $TC(i) = 0$ and solving for $e_j$ yields $e_j = \frac{(4-\lambda^2)^2}{8(1-(1-b_i-\tilde{s}_j)^2)}$, where $\tilde{s}_j$ represents the location choice that solves the constrained optimization problem. These choices of $s_j$ and $e_j$ constitute a profitable deviation if they result in a positive profit level. Substituting for this expression for $e_j$ in $S_j$’s payoff function gives

\[
\left(\frac{4-\lambda^2}{16}\right)^2 \left[ \frac{1-(b_j-\tilde{s}_j)^2}{1-(1-b_i-\tilde{s}_j)^2} \right] + \frac{3}{4} - \frac{1}{8} \left(\frac{4-\lambda^2}{1-(1-b_i-\tilde{s}_j)^2}\right)^2.
\]  

(A.4)

Redefining $S_j$’s choice variable as $\sigma = 1 - b_i - s_j$ and letting $\tilde{\sigma} = 1 - b_i - \tilde{s}_j$ yields

\[
\left(\frac{4-\lambda^2}{16}\right)^2 \left[ \frac{1-(\tilde{\sigma}-\lambda)^2}{1-(\tilde{\sigma})^2} \right] + \frac{3}{4} - \frac{1}{8} \left(\frac{4-\lambda^2}{1-(\tilde{\sigma})^2}\right)^2.
\]  

(A.5)

It is easy to verify graphically that this expression is negative for $\tilde{\sigma}, \lambda \in [0, 1]$.

Hence, there are no profitable deviations. The location and investment choices stated in Proposition 1 constitute an equilibrium.

(ii) I next show that the equilibrium stated in Proposition 1 is unique. To do this, all other possible equilibria have to be eliminated.

I start by establishing that in equilibrium, $TC(i)$ for $i = 1, 2$ never bind. To prove by contradiction, suppose not and suppose it is $TC(1)$ that binds in equilibrium. Then $S_2$ can increase its investment by $\varepsilon$ and cause a discontinuous upward jump in its payoff function. Since the cost of investment is continuous in the level of investment, there always exists a sufficiently small investment level that makes such a deviation profitable.

This result implies that I can focus on the interior solutions to the suppliers’ optimization problem as candidate equilibria. There are three types of candidate equilibria. I have already shown that one of them is an equilibrium. To eliminate the other two, it is sufficient to find a profitable deviation in each case.

Consider the candidate equilibrium where $S_i$ trades with $B_i$ for $i = 1, 2$ in the first stage of the bargaining game. The optimum choices are $s_i = b_i$ and $e_i = \frac{1}{2}$. Note that these location and investment choices imply that $TC(i)$ is satisfied. Each supplier makes $\frac{1-(1-\lambda^2)}{8}$. 

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Now suppose $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such that it serves both buyers. It must be the case that $TC(1)$ is violated and $TC(2)$ holds. $S_2$ cannot set $s_2$ and $e_2$ at their unconstrained maximum levels since doing so does not violate $TC(1)$. If it chooses $s_2$ and $e_2$ such that $TC(1)$ is just violated, it must set $e_2 = \frac{1}{(1-(1-b_1-s_2)^2)}$, where $\hat{s}_2$ represents the constrained optimal choice for location. $S_2$ makes

$$\frac{\left(1 - (b_2 - \hat{s}_2)^2\right)}{2 \left(1 - (1 - b_1 - \hat{s}_2)^2\right)} + \frac{1}{2} - \frac{(1 - \lambda^2)}{8} - \frac{1}{2 \left(1 - (1 - \hat{s}_2)^2\right)^2}. \tag{A.6}$$

Again, letting $\hat{\sigma} = 1 - b_1 - \hat{s}_2$ yields

$$\frac{\left(1 - (\hat{\sigma} - \lambda)^2\right)}{2 (1 - \hat{\sigma}^2)} + \frac{1}{2} - \frac{(1 - \lambda^2)}{8} - \frac{1}{2 (1 - \sigma^2)^2}. \tag{A.7}$$

Since it is sufficient to find only one profitable deviation, let us set $\hat{\sigma} = \frac{2}{5}$. The payoff function given in (A.6) always yields a higher value than $\frac{1-(1-\lambda^2)}{8}$ for all $\lambda \in [0,1]$.

Now consider the case when $S_i$ trades with $B_j$ for $i, j = 1, 2$ and $i \neq j$ in the second stage of the bargaining game. The optimum location and investment choices are $s_i = 1 - b_j$ and $e_i = \frac{1}{8}$. Each firm makes $\frac{1}{8}$. Suppose $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such that it serves both buyers. It sets $s_2 = b_2 + \frac{2}{5}$ and $e_2 = \frac{4-\lambda^2}{2}$. Note that $TC(1)$ is violated and $TC(2)$ is satisfied at these location and investment choices. $S_2$'s payoff if it sells to both buyers is $\frac{(4-\lambda^2)^2}{8} - \frac{1}{8}$. This is larger than $\frac{1}{8}$ for all $\lambda \in [0,1]$.

Hence, the equilibrium where $S_i$ sells to both buyers and $S_j$ sets $e_j = 0$ is the unique equilibrium.

### 3 Proof of Proposition 2

It can be shown that as $t$ increases from 0, $b^*$ changes in a monotonic way by using the implicit function theorem. Let $G(b_i, b_j; t)$ stand for the first derivative of $\pi_i = W_{B_i} - K$ with respect to $b_i$. It is equal to

$$\frac{(3 - b_i - b_j) \left(1 - (b_i - b_j)^2\right)}{8} - \frac{t (3 + b_i - b_j) (1 + 3b_i + b_j)}{18}. \tag{A.8}$$

This expression is equal to 0 at an interior solution. To solve for the symmetric equilibrium, let $b_i = b_j = b^*$. The implicit function theorem states

$$\frac{\partial b^*}{\partial t} = -\frac{\partial G(b_i, b_j; t)}{\partial t} / \frac{\partial G(b_i, b_j; t)}{\partial b^*}. \tag{A.9}$$
where the right hand side is evaluated at $b^*$. Since the second-order condition is negative, the sign of $\frac{\partial b^*/\partial t}{\partial b^*}$ is equal to the sign of the numerator. The numerator is

$$\frac{\partial G(b_i, b_j; t)}{\partial t} = \frac{-(3 + b_i - b_j)(1 + 3b_i + b_j)}{18} < 0.$$  \hspace{1cm} (A.10)

Hence, $b^*$ is a decreasing function of $t$. As $t$ increases from 0, $b^*$ decreases from $1/2$ to 0 in a continuous fashion.

To determine the exact parameter values for which the buyers will be less than maximally differentiated, note that the analysis so far is based on the assumptions that $K$ is low enough for both buyers to produce positive quantities, and that $K$ is higher than the cost reduction obtained from the inputs provided by the suppliers. Hence, it remains to show that there exist $t$ values for which both of these assumptions are satisfied and the buyers choose intermediate locations. The two assumptions imply that

$$\frac{(4 - \lambda^2)^2}{16} \leq K \leq t \frac{\lambda(3 + b_i - b_j)^2}{18} + \frac{(4 - \lambda^2)^2}{32}, \hspace{1cm} (A.11)$$

where the first term stands for the benefit from an input and the last two terms represent the profits in the product market and the net benefit from an input (after making the input payment) respectively. It is straightforward to verify that $b^*$ is positive for $t < 2.25$. Evaluating (A.11) at $b^*$ and numerically solving for $t$ reveals that the lower limit on $K$ is smaller than the upper limit for $t > 1.13$, which implies that for these $t$ values, there exist $K$ values which satisfy the inequalities given in (A.11). Hence, for $1.13 < t < 2.25$, the buyers choose intermediate locations.\footnote{Obviously, the exact values given for $t$ are highly sensitive to the model specification. It can be easily verified that the range of $t$ values for which the buyers choose intermediate locations gets larger with sufficiently small downward shifts in the investment cost function or sufficiently small upward shifts in the input benefit function such that there is still only one supplier in the market in equilibrium.}

4 Proof of Proposition 3

There are three types of candidate equilibria. In the following claims, I consider each in turn.

Claim 1 If $\lambda < \hat{\lambda} = \sqrt{\frac{1 - 2\sqrt{\gamma}}{\gamma}}$, $S_i$ locates in the middle between the two buyers and sells inputs to both. $e_i = 1 - \frac{3\lambda^2}{4}$ while $e_j = 0$.

Proof. The value of the input to both buyers is positive if $\lambda < \frac{1}{\sqrt{\gamma}}$. This condition is automatically satisfied for all $\lambda < \hat{\lambda}$. Can supplier $S_i$ do better by deviating? Deviating

\footnote{16}
and locating at the same point as one of the buyers may be preferable for sufficiently large λ values. If \( S_i \) serves one of the buyers only, its profit-maximizing location and investment choices are \( s_i = b_i \) and \( e_i = \frac{1}{2} \). Such a deviation may be more profitable only in the case when \( B_j \) does not find it profitable to use \( S_i \)’s input when \( S_i \) locates at \( s_i = b_i \). Otherwise, the supplier’s profit-maximizing options are as given above.

The payoff to \( S_i \) from deviating and serving one of the buyers only is \( \frac{1}{8} \). Comparing this with \( S_i \)’s equilibrium payoff yields that such a deviation is profitable if and only if \( \lambda > \sqrt{\frac{2}{7}} \). Hence, for \( \lambda < \hat{\lambda} \), \( S_i \) does not have a profitable deviation.

Consider now whether supplier \( S_j \) can do better by deviating. It makes 0 in equilibrium. It is necessary to check whether it can earn a positive profit by setting \( s_j \) and \( e_j \) such that it trades with \( B_j \) only, with \( B_i \) only, or with both of the buyers. If it trades with \( B_j \), it makes

\[
\frac{1}{2} e_j \left( 1 - \gamma (b_j - s_j)^2 \right) - \frac{1}{2} e_i \left( 1 - \gamma (1 - b_j - s_i)^2 \right) - \frac{e_j^2}{2}.
\]

The unconstrained optimum is \( s_j = b_j \) and \( e_j = \frac{1}{2} \). For \( B_j \) to prefer to trade with \( S_j \), \( TC(j) \) must be holding. Given the equilibrium strategy of \( S_i \), this condition is satisfied. Substituting for the relevant choices of \( s_i, s_j, e_i \) and \( e_j \) yields \( \frac{1}{8} - \frac{(4 - \gamma \lambda^2)^2}{64} \), which is negative for \( \lambda < \hat{\lambda} = \sqrt{\frac{4 - 2\sqrt{2}}{7}} \). Hence, this is not a profitable deviation.

If it trades with \( B_i \), it makes

\[
\frac{1}{2} e_j \left( 1 - \gamma (1 - b_i - s_j)^2 \right) - \frac{e_j^2}{2}.
\]

The location and investment choices that maximize this payoff function are \( s_j = 1 - b_i \) and \( e_j = \frac{1}{2} \). For \( B_i \) to prefer to buy from \( S_j \) instead of from \( S_i \), it must be the case that \( TC(i) \) is violated. Given the equilibrium strategy of \( S_i \), \( TC(i) \) holds at the unconstrained optimum stated above for \( \lambda < \hat{\lambda} = \sqrt{\frac{4 - 2\sqrt{2}}{7}} \). Supplier \( S_j \) can choose \( s_j \) and \( e_j \) such that \( TC(i) \) is just violated. The constrained optimal choices are \( s_j = 1 - b_i \) and \( e_j = \frac{(4 - \gamma \lambda^2)^2}{8} \). Evaluating \( S_j \)'s profit level at these choices yields

\[
\left( \frac{(4 - \gamma \lambda^2)^2}{16} \right) \left( 8 - \frac{(4 - \gamma \lambda^2)^2}{8} \right).
\]

Again, since this expression is negative for \( \lambda < \hat{\lambda} = \sqrt{\frac{4 - 2\sqrt{2}}{7}} \), this is not a profitable deviation.

Finally, \( S_j \) can set \( s_j \) and \( e_j \) such that both buyers prefer to trade with it. This implies that its choices must violate \( TC(i) \) and satisfy \( TC(j) \). \( S_j \)'s payoff is given by

\[
\frac{1}{2} \left[ e_j \left( 2 - \gamma (b_j - s_j)^2 - \gamma (1 - b_i - s_j)^2 \right) - \frac{1}{2} e_i \left( 1 - \gamma (1 - b_j - s_i)^2 \right) \right] - \frac{e_j^2}{2}.
\]
If $TC (i)$ and $TC (j)$ are evaluated at the unconstrained optimum of this payoff function, both of them are satisfied, which implies that $B_i$ prefers to trade with $S_i$ instead of $S_j$. Hence, consider values of $s_i$ and $e_i$ such that $TC (i)$ is just violated. Note that if there is not a profitable deviation that satisfies only one of the constraints, there cannot be a profitable deviation that satisfies both of the constraints. Setting $TC (i) = 0$ and solving for $e_j$ yields $e_j = \frac{(4-\gamma \lambda^2)^2}{8(1-\gamma(1-b_i-s_j)^2)}$, where $\tilde{s}_j$ represents the location choice that solves the constrained optimization problem. These choices of $s_j$ and $e_j$ constitute a profitable deviation if they result in a positive profit level. Substituting for the expressions for $e_j, e_i$ and $s_i$ in $S_j$’s payoff function yields

$$\frac{(4-\gamma \lambda^2)^2}{16} \left[ \frac{1-\gamma(b_j-\tilde{s}_j)^2}{(1-\gamma(1-b_j-\tilde{s}_j)^2)} + \frac{3}{4} - \frac{(4-\gamma \lambda^2)^2}{8(1-\gamma(1-b_j-\tilde{s}_j)^2)^2} \right].$$  \hspace{1cm} (A.16)

Redefining $S_j$’s choice variable as $\sigma = 1 - b_i - s_j$ and letting $\tilde{\sigma} = 1 - b_i - \tilde{s}_j$ generates

$$\frac{(4-\gamma \lambda^2)^2}{16} \left[ \frac{1-\gamma \lambda^2(1-\tilde{\sigma}^2)}{(1-\gamma \lambda^2(\tilde{\sigma}^2)^2)} + \frac{3}{4} - \frac{(4-\gamma \lambda^2)^2}{8(1-\gamma(1-b_j-\tilde{s}_j)^2)^2} \right],$$  \hspace{1cm} (A.17)

where $\tilde{\sigma}$ represents the ratio of the distance between $S_j$ and $B_i$ to the distance between $B_i$ and $B_j$. Since $S_j$ deviates to serve both buyers, its optimal location is between the two buyers. This implies that $\tilde{\sigma}$ ranges between 0 and 1. Substituting for $\gamma \lambda^2 = \theta$ yields

$$\frac{(4-\theta)^2}{16} \left[ \frac{1-\theta (1-\tilde{\sigma}^2)}{(1-\theta(\tilde{\sigma}^2)^2)} + \frac{3}{4} - \frac{(4-\theta)^2}{8(1-\theta(\tilde{\sigma}^2)^2)^2} \right],$$  \hspace{1cm} (A.18)

To prove the claim, it is necessary to check whether this expression is negative for $\tilde{\sigma} \in [0, 1]$ and $\theta \in [0, 4 - 2\sqrt{2}]$. It is easy to verify graphically that this is the case. Hence, for $\lambda < \tilde{\lambda}$, there are no profitable deviations from the equilibrium where $S_i$ sells inputs to both buyers by choosing $s_i = b_i + \frac{\lambda}{2}$ and $e_i = 1 - \frac{2\lambda^2}{4}$, and $S_j$ invests zero.

Claim 2 If $\lambda > \tilde{\lambda}$ where $0 < \tilde{\lambda} < \tilde{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$, $S_i$ trades with $B_i$ for $i = 1, 2$ in the first stage of the bargaining game. The optimal choices are $s_i = b_i$ and $e_i = \frac{1}{2}$.

Proof. Since $TC (i)$ is satisfied at the equilibrium location and investment choices, each supplier makes $\frac{1}{8} - \max \left\{ 0, \frac{(1-\gamma \lambda^2)}{8} \right\}$ in equilibrium. To see that there are no profitable
deviations for $\lambda > \tilde{\lambda}$, suppose $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such that it serves both buyers.\footnote{Note that as long as $(1 - \gamma \lambda^2) > 0$, deviating and serving $B_1$ only is not a feasible option since $B_2$ would continue to buy from $S_2$. On the other hand, if $(1 - \gamma \lambda^2) < 0$, $S_2$ cannot earn a higher payoff by deviating and trying to serve $B_1$ given the high amount of investment it would have to make to have $TC(1)$ violated.} It must be the case that $TC(1)$ is violated and $TC(2)$ holds. $S_2$ cannot set $s_2$ and $e_2$ at their unconstrained maximum levels since doing so does not violate $TC(1)$. If it chooses $s_2$ and $e_2$ such that $TC(1)$ is just violated, the effort level is given by $e_2 = \frac{1}{(1 - \gamma (1 - b_1 - \tilde{s}_2)^2)}$, where $\tilde{s}_2$ represents the constrained optimal location choice. $S_2$ makes

$$\frac{1 - \gamma (b_2 - \tilde{s}_2)^2}{2 (1 - \gamma (1 - b_1 - \tilde{s}_2)^2)} + \frac{1}{2} - \max \left\{ 0, \frac{1 - \gamma \lambda^2}{8} \right\} - \frac{1}{2} \frac{1 - \gamma (1 - b_1 - \tilde{s}_2)^2}{(1 - \gamma (1 - b_1 - \tilde{s}_2)^2)^2}. \tag{A.19}$$

After redefining $S_2$’s choice variable as $\sigma = 1 - b_1 - s_2$ and letting $\tilde{\sigma} = 1 - b_1 - \tilde{s}_2$, the net benefit from deviating is given by

$$\Delta = \frac{1 - \gamma (\tilde{\sigma} - \lambda)^2}{2 (1 - \gamma \tilde{\sigma}^2)} + \frac{3}{8} - \frac{1}{8} \frac{1 - \gamma \lambda^2}{2 (1 - \gamma \tilde{\sigma}^2)^2}. \tag{A.20}$$

By the envelope theorem, $\frac{\partial \Delta}{\partial \lambda} = 2 \gamma (\tilde{\sigma} - \lambda) < 0$ since $\tilde{\sigma} < \lambda$. If $\lambda = 0$, $\Delta > 0$ since $\tilde{\sigma} = 0$. If $\lambda = \hat{\lambda} = \sqrt{\frac{1 - 2 \gamma \tilde{\sigma}^2}{\gamma}}$, it is straightforward to verify that $\Delta < 0$ by substituting for $\gamma = \frac{4 - 2 \gamma \tilde{\sigma}^2}{\lambda^2}$ in (A.20) and evaluating the resulting expression graphically for $\frac{2}{\lambda} \in [0, 1]$. Hence, there must exist a critical $\lambda = \hat{\lambda} \in \left(0, \hat{\lambda}\right)$ such that for $\lambda > \tilde{\lambda}$, the candidate equilibrium is an equilibrium and for $\lambda < \hat{\lambda}$, it is not. $\blacksquare$

**Claim 3** If $\lambda > \tilde{\lambda} = \sqrt{\frac{1 - 2 \gamma \tilde{\sigma}^2}{\gamma}}$, $S_i$ trades with $B_j$ for $i, j = 1, 2$ and $i \neq j$ in the second stage of the bargaining game. The optimal choices are $s_i = 1 - b_j$ and $e_i = 1/2$.\footnote{Note that deviating and serving $B_2$ only cannot be a profitable deviation since $S_2$ can capture a greater share of the surplus created if it trades with $B_1$ in the second stage of the bargaining game. This is because $S_2$ would be negotiating with $B_2$ in the first stage of the bargaining game when $B_2$ has the option of negotiating with $S_1$ if the negotiation with $S_2$ fails. In contrast, if the negotiation between $S_2$ and $B_1$ fails, $B_1$ does not have access to an alternative supply of inputs.}

**Proof.** Each firm makes $\frac{1}{3}$ in equilibrium. Both $TC(1)$ and $TC(2)$ have to be violated for this equilibrium to exist. This requires $\lambda > \frac{1}{\sqrt{2}}$. To see that there are no profitable deviations for $\lambda > \tilde{\lambda}$, suppose $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such that it serves both buyers.\footnote{Note that as long as $(1 - \gamma \lambda^2) > 0$, deviating and serving $B_1$ only is not a feasible option since $B_2$ would continue to buy from $S_2$. On the other hand, if $(1 - \gamma \lambda^2) < 0$, $S_2$ cannot earn a higher payoff by deviating and trying to serve $B_1$ given the high amount of investment it would have to make to have $TC(1)$ violated.} It sets $s_2 = b_2 + \frac{1}{2}$ and $e_2 = 1 - \frac{\gamma \lambda^2}{4}$. Note that $TC(1)$ is always violated and $TC(2)$ is satisfied for $\lambda > \sqrt{\frac{2}{\gamma}}$ at these location and investment
choices. $S_2$’s payoff if it sells to both buyers is $\frac{(4-\gamma\lambda^2)^2}{32} - \frac{1}{2}$. This implies that the net benefit from deviating is $\geq 0$ for $\lambda < \hat{\lambda} = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$.

The three claims jointly imply the results stated in Proposition 3.

5 Proof of Corollary 1

It is shown in the proof of Proposition 3 that $\hat{\lambda}(\gamma) = \sqrt{\frac{4-2\sqrt{2}}{\gamma}}$. This is clearly decreasing in $\gamma$. To see that $\hat{\lambda}(\gamma)$ is also decreasing in $\gamma$, note that it is defined by $\Delta = 0$, where $\Delta$ is given by (A.20). Hence, by the implicit function theorem,

$$\frac{\partial \hat{\lambda}}{\partial \gamma} = -\frac{\partial \Delta/\partial \gamma}{\partial \Delta/\partial \hat{\lambda}}.$$ 

In the denominator, $\frac{\partial \Delta}{\partial \lambda} = 2\gamma (\hat{\sigma} - \lambda) < 0$ since $\hat{\sigma} < \lambda$. The expression in the numerator is also negative since

$$2\left(\frac{1}{2}(1-\gamma\hat{\sigma})^2 - 1\right) < 0.$$

Hence, $\hat{\lambda}(\gamma)$ is decreasing in $\gamma$.

6 Proof of Lemma 2

Consider the SPNE outcome where $B_i$ buys inputs from $S_i$ for $i = 1, 2$. This implies $s_i = b_i$. If $B_i$ were to use $S_j$’s input instead, the benefit would be $e_j (1 - \gamma\lambda^2)$. This expression is negative for $\lambda > \frac{1}{\sqrt{2}}$. Proposition 3 states that there can be two suppliers in equilibrium for $\lambda > \tilde{\lambda}$. Hence, to prove the result stated in the lemma, it is necessary to show that $\tilde{\lambda} > \frac{1}{\sqrt{2}}$.

Suppose not. Then neither supplier should have a profitable deviation at $\lambda = \frac{1}{\sqrt{2}}$. Suppose, as in the proof of Proposition 3, $S_1$ plays its equilibrium strategy and $S_2$ deviates by choosing $s_2$ and $e_2$ such that it serves both buyers. $S_2$’s net benefit from deviating is given by (A.20), where $\hat{\sigma} = 1 - b_1 - \hat{s}_2$ stands for the distance between $S_2$’s optimal location and $B_1$. Note that since $S_2$ deviates from its equilibrium strategy to serve both buyers, its optimal location must be between the two buyers. Hence, after substituting for $\gamma = \frac{1}{\sqrt{2}}$, it is straightforward to see that for a given $\lambda$ value, the supplier chooses a value of $\frac{\hat{\sigma}}{\lambda} \in [0, 1]$. Since it is sufficient to find only one profitable deviation, consider $\frac{\hat{\sigma}}{\lambda} = 0.4$. The net benefit from deviating is positive at $\frac{\hat{\sigma}}{\lambda} = 0.4$ and, hence, it must be the case that $\tilde{\lambda} > \frac{1}{\sqrt{2}}$.  

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7 Proof of Proposition 4

(i) Suppose $\tilde{\lambda} (\gamma) > 1$. Proposition 3 states that for all $\lambda < \tilde{\lambda} (\gamma)$, only one supplier invests a positive amount and locates between the two buyers. Proposition 2 implies that in such cases the buyers may choose intermediate locations for sufficiently small values of $t$.

(ii) Suppose $\tilde{\lambda} (\gamma) < 1$. I show that there exist values of $t$ and $\gamma$ such that the following strategies constitute an equilibrium. In the first stage of the game, buyer $B_i$ chooses $b_i = 0$ for $i = 1, 2$. In the second stage, supplier $S_i$ chooses $s_i = b_i$ and $e_i = \frac{1}{2}$ for $i = 1, 2$.

Consider $\gamma$ sufficiently high such that $\tilde{\lambda} (\gamma) = \sqrt{\frac{4 - 2\sqrt{2}}{\gamma}} < 1$. By Proposition 3, for all $\lambda$ values such that $\tilde{\lambda} < \lambda \leq 1$, there are two suppliers making positive investments in equilibrium. Does either of the buyers have an incentive to unilaterally deviate? Suppose $B_i$ chooses $b_i > 0$ such that in the second stage of the game, both suppliers still make positive investments and produce specialized inputs. Lemma 2 implies that $B_i$’s payoff is given by

$$\pi_i = t \frac{\lambda(3 + b_i - b_j)^2}{18} - K + \frac{1}{8}. \quad (A.21)$$

which is strictly decreasing in $b_i$. Hence, this cannot be a profitable deviation.

Now suppose $B_i$ chooses $b_i > 0$ such that in the second stage of the game, only one supplier invests a positive amount and produces a non-specialized input. I next show that such a deviation would not be profitable for sufficiently high values of $t$. To see this, first note that Proposition 2 implies that for sufficiently high values of $t$, $B_i$’s unconstrained location choice is $b_i = 0$ even if there will be only one non-specialized supplier in the market. Hence, by choosing $b_i > 0$, $B_i$ must be achieving a lower payoff than it would achieve at $b_i = 0$. Second, the following lemma implies that for $\tilde{\lambda} < 1$, $B_i$’s payoff at $\lambda = 1$ (i.e., when $b_i = b_j = 0$) is higher if there are two specialized suppliers in the market than if there is a single non-specialized supplier.

Lemma 3 For a given value of $\lambda$, let $\pi^1_i$ and $\pi^2_i$ stand for $B_i$’s payoff when the buyers are served by a single non-specialized supplier and two fully specialized suppliers, respectively. $\pi^1_i \leq \pi^2_i$ for $\lambda \geq \tilde{\lambda}$.

Proof. It is shown in Proposition 3 that if there is a single non-specialized supplier in the market, the value of the input to both buyers is positive if $\lambda < \frac{2}{\sqrt{\gamma}}$. Suppose this condition holds, which implies that

$$\pi^1_i = t \frac{\lambda(3 + b_i - b_j)^2}{18} - K + \frac{1}{4} \left(1 - \gamma \frac{\lambda^2}{4}\right)^2. \quad (A.22)$$
Lemma 2 implies that if there are two specialized suppliers in the market, \( B_i \)'s payoff is given by (A.21). Subtracting \( \pi_1^i \) from \( \pi_2^i \) yields \( \frac{1}{8} - \frac{1}{4} \left(1 - \gamma \frac{\lambda^2}{\gamma} \right)^2 \), which is < 0 for \( \lambda = 0, = 0 \) for \( \lambda = \tilde{\lambda} \), and is increasing in \( \lambda \) for all \( \lambda < \frac{2}{\sqrt{7}} \). Hence, it is negative for all \( \lambda < \tilde{\lambda} \) and positive for all \( \tilde{\lambda} < \lambda < \frac{2}{\sqrt{7}} \). For \( \lambda > \frac{2}{\sqrt{7}} \), it is trivially the case that \( \pi_2^i > \pi_1^i \) since

\[
\pi_1^i = t \frac{\lambda(3 + b_i - b_j)^2}{18} - K. \tag{A.23}
\]

Hence, choosing \( b_i > 0 \) such that the buyers are served by a single non-specialized supplier must yield a strictly lower payoff than the equilibrium payoff level. I conclude that for sufficiently high values of \( t \) and \( \gamma \), there are no incentives to unilaterally deviate from the equilibrium strategies stated above.

(iii) Finally, I show that there is no SPNE where there are two fully specialized suppliers in the market and the buyers choose intermediate locations. Suppose not. That is, suppose that there is a SPNE where there are two fully specialized suppliers in the market and the buyers choose intermediate locations (i.e., \( b_i > 0 \) for \( i = 1, 2 \)). From Proposition 3, it must be the case that \( \lambda > \tilde{\lambda} \) or else one of the suppliers would have incentives to deviate. Lemma 2 implies that \( B_i \)'s payoff is given by (A.21).

To see whether there are any profitable deviations, suppose \( B_i \) marginally decreases \( b_i \). Depending on the magnitude of \( \lambda \) at this candidate equilibrium and on the suppliers' strategies, if \( B_i \) makes such a deviation, the buyers may either be served by a single non-specialized supplier or they may continue to be served by two fully specialized suppliers. Suppose the buyers continue to be served by two fully specialized suppliers. Since (A.21) is decreasing in \( b_i \), such a deviation must strictly increase \( B_i \)'s payoff. Now suppose that marginally decreasing \( b_i \) leads to the buyers being served by a single supplier. Note that a necessary condition for this outcome is \( \tilde{\lambda} \leq \lambda < \tilde{\lambda} \). By Lemma 3, this deviation must also strictly increase \( B_i \)'s payoff.

Hence, regardless of the suppliers' strategies, there is always a profitable deviation from the candidate equilibrium where there are two fully specialized suppliers in the market and the buyers choose intermediate locations.