Optimal Technology Sharing Strategies in Dynamic Games of R&D[1]

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Abstract

A question central to R&D policy making is the impact of competition on cooperation. This paper builds a theoretical foundation for the dynamics of knowledge sharing in private industry. We model an uncertain research process and ask how the incentives to license intermediate steps to rivals change over time as the research project approaches maturity. Such a dynamic approach allows us to analyze the interaction between how close the firms are to product market competition and how intense that competition is. We uncover a basic dynamic of sharing such that firms are less likely to share as they approach the product market. This dynamic is driven by a trade-off between three effects: the rivalry effect, the duplication effect and the speed effect. We show that this dynamic can be reversed when duopoly profits are sufficiently low or when the firms have asymmetric research capabilities. We also explore the implications of the model for patent policy, and compare policies targeting early research outcomes with policies targeting late research outcomes.

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1 Introduction

This paper builds a theoretical foundation for the dynamics of knowledge sharing in private industry. The substantial evidence on licensing, research alliances and joint ventures reveals that knowledge sharing arrangements are a central way in which firms acquire technological knowledge. From a social welfare perspective, sharing of research outcomes is desirable because it results in less duplication. Since the 1980s, governments in the US and Europe have actively promoted joint R&D projects through subsidies, tolerant antitrust treatment, and government-industry partnerships.[1] At the same time, economics research has studied the private and social incentives to have knowledge sharing arrangements, focusing on issues of appropriability and spillovers. However, none of these studies has focused on the basic dynamics of private sharing incentives. Research projects in industries such as biotechnology, automobiles and computers can take years or even decades to complete. Over such long time horizons, firms may decide to share some intermediate steps, but not all of their research outcomes. Consider, for example, the collaboration between GM and Toyota to develop fuel cell technology for automobiles. In 2006, after more than 6 years of working together, the two companies ended their collaboration because they could no longer agree to terms for sharing intellectual property.[2]

Focusing on the dynamics of research, we ask how the incentives to license research outcomes to rivals change over time as a research project approaches maturity. A question central to the policy debate, as well as the study of knowledge sharing arrangements, is the impact of competition on cooperation. This is because in many cases, the most suitable research partner for a firm may be one of its competitors. However, as in the case of GM and Toyota, such sharing poses especially difficult challenges because it may reduce the commercial value of the

[1] For example, in the US, the National Cooperative Research and Production Act (NCRPA) of 1993 provides that research and production joint ventures be subject to a 'rule of reason' analysis instead of a per se prohibition in antitrust litigation. In the EU, the Commission Regulation (EC) No 2659/2000 (the EU Regulation) provides for a block exemption from antitrust laws for RJVs, provided that they satisfy certain market share restrictions and allow all joint venture participants to access the outcomes of the research.

[2] See "GM and Toyota end collaboration on fuel cells" at http://www.businessrespect.net/page.php?Story_ID=1537. As another example, consider alliances in biotechnology. Lerner and Merges (1998) find that while in a few cases the alliances covered technologies well along the way to regulatory approval, in most cases they were arranged at the earliest stages of research (prior to animal studies, clinical trials, and regulatory approval).

firms' R&D efforts.[3] A dynamic perspective allows us to analyze the impact of competition on cooperation in two different ways. We can analyze the impact of both how close the firms are to product market competition and how intense that competition is. Our results reveal an interesting interaction between these two factors.

From a dynamic perspective, the process of research is generally characterized by a high level of uncertainty in the beginning. For example, at the outset of research on a new medical drug, the expected success rate may be as low as 2% and the expected time to market may be more than a decade.[4] Similarly, fuel cell technology for automobiles has been in active development since the 1990s and is not expected to reach full commercial viability for another decade.[5] In such environments, progress in research can be described as a decrease in the level of uncertainty that researchers face. One of the novel aspects of this paper is to analyze how firms' incentives to share research outcomes change during a research process as the level of uncertainty they face decreases. We show that the impact of uncertainty on firms' sharing incentives depends on the intensity of product market competition.

We assume that research projects consist of two sequential steps. Researchers cannot earn any profits before completing both steps of the project. An important feature of the model is that the research steps are symmetric in all respects except in regards to how far away they are from the end of the project. We deliberately assume that there are no spillovers in research in order to focus on the role uncertainty plays in knowledge sharing. It has been stressed in the literature that firms may have higher spillover rates and bigger appropriability problems in earlier stages of research than in later stages of research. Although the rate of spillovers may shape the dynamics of sharing, our results show that this is not the only factor that matters.

We assume that firms are informed about the progress of their rivals and make joint sharing decisions after each success. The leading firm sets a licensing fee which is paid by the lagging

[3] Empirical evidence suggests that firms do take measures to avoid opportunistic behavior when they are collaborating with their competitors. For example, Oxley and Sampson (2004) show that direct competitors choose to limit the scope of alliance activities. Majewski (2004) shows that direct competitors are more likely to outsource their collaborative R&D.

[4] See Northrup (2005).

[5] For a timeline, see "Fuel Cell Electric Vehicles: The Road Ahead" available at http://www.fuelcelltoday.com/media/1711108/fuel_cell_electric_vehicles_-_the_road_ahead.pdf.

firm. We identify three effects which shape the sharing decisions. Sharing has the benefits of avoiding the duplication of R&D costs (duplication effect) and bringing product market profits forward (speed effect). However, on the negative side, joint product market profits may be reduced as a result of sharing since sharing decreases the leader's ability to earn monopoly profits (rivalry effect).

Our main result for symmetric firms reveals a basic dynamic of sharing such that the firms are less likely to share as they approach the product market. That is, the incentives to share research outcomes decrease with progress. This is so even though licensing fees may increase with progress. We show that this dynamic arises whenever duopoly profits are so high that a lagging firm stays in the race no matter how far behind it is. The reason for the decline in the sharing incentives is that earlier in the race, the speed effect is more important relative to the rivalry effect. We show that this is tied to the resolution of uncertainty. Since the leader's chance of finishing first is lower earlier in the race, the importance of the rivalry effect is diminished. Intuitively, there is less value in maintaining a lead when it is more likely to be lost in the future.

That sharing incentives decrease with progress makes intuitive sense because it shows that as the firms get closer to the end of the R&D race, the impending competition harms cooperation.[6] However, we show that when duopoly profits are too low to keep lagging firms in the race, sharing incentives may increase with progress rather than decrease. This is because a lagging firm is most likely to exit early in the race when it has not made much progress. Thus, the firms may decide against sharing early on to take advantage of this.

Sharing incentives may also increase if the firms differ in their research capabilities (such as in the biotechnology industry). With asymmetric firms, there is an additional effect which

[6] For example, the collaboration between GM and Toyota on fuel cell technology for automobiles lasted for more than 6 years before their competitive rivalry made cooperation too difficult. A spokesman for GM, Scott Fosgard, reportedly said that "the companies will no longer collaborate on fuel cells because that technology is moving out of the research stage and into the more proprietary development stage. But both companies remain open to other research projects in mutually beneficial areas." See "GM, Toyota end joint fuel cell research" at http://www.msnbc.msn.com/id/11654151/ns/business-oil_and_energy/t/gm-toyota-end-joint-fuel-cell-research/#.UNJvFMXNm0Y. The collaboration on fuel cell technology between Daimler and Ford has proved longer lasting, perhaps because the rivalry between these two firms is less intense. See Steinemann (1999) and http://cafcp.org/about-us/members/automotive-fuel-cell-cooperation.

may shape the sharing incentives. We show that if one of the firms has increasing research costs over time, sharing may cause a firm to start working on a more costly research step. This progress effect may cause sharing incentives to increase over time.

These results have implications for policy making in innovation environments. They show that the design of optimal policies should be sensitive to the dynamic sharing patterns which would emerge in the absence of such policies. Our results emphasize that the dynamics of sharing may be driven by the intensity of product market competition.

Specifically, we consider how patent policy can be used to change the sharing incentives in our dynamic framework. Intuition suggests that broader patent protection should encourage sharing by increasing the cost of duplication.[7] We show that in a dynamic framework, this may not always be the case. Broader patent protection of late stage research may feed back to discourage sharing of early stage research. This is due to a progress effect similar to the one discussed above for the case of asymmetric firms. This raises the question of whether it is more desirable to have sharing of early or late research outcomes. We show that encouraging sharing of late research outcomes at the expense of discouraging sharing of early research outcomes would generally be undesirable.

The impact of competition on cooperation in R&D has been the focus of many papers in the economics literature. These papers have mainly studied firms' incentives to share research outcomes at one point in time, either before the start of research, as in the case of research joint ventures, or after the development of a technology, as in the case of licensing.[8] A general result in these papers is that as product market competition increases, incentives to cooperate decrease.[9] We contribute to this literature by focusing on the interaction between competition and cooperation in a dynamic setting.

[7] It has also been stressed in the literature that patents can facilitate the transfer of technologies through licensing because they increase the bargaining power of the licensor. See, for example, the discussion in Gallini (2002). Anand and Khanna (2000), and Arora and Fosfuri (2000) show empirically that licensing is more prevalent in industries where patent protection is more effective.

[8] See, for example, Kamien (1992) on licensing, and D'Aspremont and Jacquemin (1988) on research joint ventures. Severinov (2001) studies the impact of product market competition on the incentives for informal sharing of R&D outcomes between employees.

[9] For example, Choi (1993) shows that competing firms will cooperate if the level of spillovers are sufficiently high. Wang (2002) shows that licensing between competitors will take place if they produce sufficiently differentiated products.

Our paper is related to the literature which models R&D as a multi-stage process.[10] Grossman and Shapiro (1987) and Harris and Vickers (1987) analyze how firms vary their research efforts over the course of a research project.[11] A common assumption that has been made in this literature is that the different stages of R&D differ from each other in a fundamental way. For example, Reinganum (1985) considers a model with a research and a development stage, and assumes that the findings in the first stage rapidly become public knowledge (see also Vonortas, 1994). Aghion, Dewatripont and Stein (2008), and Cozzi and Galli (2011) assume that different stages of research are conducted by different institutional players, namely academia and private firms, which is why there may be more dissemination of research outcomes early on (see also Hellmann and Perotti, 2011). Although declining sharing incentives is also one of our results, it happens for very different reasons in our model with symmetric firms.

Disclosure of intermediate research outcomes has also been considered in Scotchmer and Green (1990), d'Aspremont et al. (2000), Bar (2006), Bessen and Maskin (2009), and Fershtman and Markovich (2010). Scotchmer and Green (1990) consider disclosure through patenting while Bar (2006) studies disclosure through publishing. Similar to our paper, d'Aspremont et al. (2000), Bessen and Maskin (2009), and Fershtman and Markovich (2010) consider licensing of intermediate research outcomes. However, none of these papers focus on the dynamics of sharing incentives.

The paper proceeds as follows. In the next section, we describe the set-up and explain, as a benchmark, what happens if the firms are allowed to collude in the product market. In section 3, we analyze the effect of competition on the dynamic sharing incentives of symmetric firms. In sections 4 and 5, we consider extensions of our basic framework to the case when duopoly profits are too low to keep lagging firms in the race and the case of asymmetric firms. After analyzing the impact of patent policy on the sharing incentives in our dynamic set-up in section 6, we conclude in section 7.

[10] An important issue in this literature is whether the different stages of R&D are carried out by the same player or by different players. If they are carried out by different players, this raises the question of how the rents should be distributed between the different generations of innovators. See, for example, Scotchmer (1991).

[11] See also Cabral (2003) and Judd (2003) who analyze the risk-taking behavior of firms.

2 Model

Since we are interested in the effect of competition on firms' incentives to share, we consider an environment with two firms, $i = 1, 2$, which invest in a research project. On completion of the project, a firm can produce output in a product market. We consider Markov Perfect Equilibria (MPE), where each firm maximizes its discounted expected continuation payoff given the Markov strategy of the other firm. Before describing the payoffs and the MPE, we first explain the research and production phases.

2.1 Research Environment

To capture the idea of progress, we consider a research project with 2 distinct steps. These steps may be thought of as early and late stage research. There is no difference between the steps in terms of the technology or the options available to the firms. This is because we seek to derive endogenous differences between the research steps that result from the dynamics in the decisions made by the firms. A firm cannot start to work on the next step before completing the prior step, and all steps of the project need to be completed successfully before a firm can produce output.

We assume that each firm operates an independent research facility. We model research activity using a Poisson discovery process. Time is continuous, and the firms share a common discount rate $0 < r < 1$. Following Lee and Wilde (1980), we assume that to conduct research, a firm must incur a flow cost c per unit of time.[12] Investment provides a stochastic time of success that is exponentially distributed with hazard rate $\alpha > 0$. A higher value of α corresponds to a shorter expected time to completion. For a firm which has not yet completed the project, a decision not to invest the flow cost c is assumed to be irreversible and equivalent to dropping out of the game.

When one firm successfully completes a stage of research before the other firm does, we

[12] In the concluding section of the paper, we discuss how the results extend to a model with continuous effort choices. The discrete effort assumption can be motivated by presuming a fixed amount of effort that each firm can exert, which is determined by the capacity of its R&D division. As an example, Khanna and Iansiti (1997) explain that given the highly specialized nature of the R&D involved in designing state-of-the-art mainframe computers, firms in this industry find it very expensive to increase the number of researchers available to them.

assume that the leading firm can share this knowledge with the lagging firm and thereby save the lagging firm from having to continue to invest to complete the stage. There are a variety of ways to model the sharing process. We consider ex post sharing or licensing, where the leading firm shares its result with the lagging firm in exchange for a licensing fee.

Regarding the information structure, we assume that the lagging firm cannot observe the technical content of the rival's research without explicit sharing.[13] In this sense, there are no technological spillovers. Everything else in the game is common knowledge. In particular, firms observe whether their rival is conducting research as well as whether the rival has a success. Third parties such as courts also observe this information and can enforce the licensing contracts.

2.2 Product Market Competition

We represent the product market competition in the following reduced form way. If both firms have completed the research project, they compete as duopolists and each earns a flow profit of π^D forever. If only one firm has completed the research project, the firm earns a monopoly flow profit of π^M as long as the other firm does not produce output. Here, $\pi^M > \pi^D$. As a benchmark, we will consider the case that the firms make production decisions to maximize their joint profits in the product market. This results in a joint flow profit of π^J. We assume that the magnitudes of π^D, π^M and π^J do not depend on the decisions taken during the research phase.

These payoffs are sufficiently flexible to capture various models of product competition.[14] For example, if the firms produce homogeneous products and compete as Bertrand or Cournot competitors, then $\pi^J = \pi^M > 2\pi^D$. If the firms produce differentiated products, then $\pi^J > \max\left\{\pi^M, 2\pi^D\right\}$ and the relationship between π^M and $2\pi^D$ will depend on the degree of product differentiation. For low levels of product differentiation, $\pi^M > 2\pi^D$; for high levels of product

[13] Alternatively, we could assume that research results can be copied, but successful firms win immediate patents. A leading firm could then prevent a lagging firm from copying its research by enforcing its patent. If the patent does not prevent the rival from developing a non-infringing technology at the same flow cost c and with the same hazard rate, then the formal set up would be equivalent to ours. We consider the case when patenting changes the research cost of the lagging firm in section 6.

[14] We assume that the firms conduct the research to solve the same technical problem. However, unmodelled differences in production technologies can still lead them to produce differentiated products.

differentiation, $\pi^M \leq 2\pi^D$.[15]

In the research phase of the model, firms make both sharing and investment decisions. In our basic analysis, we assume that firms always invest. This allows us to study how the sharing dynamics depend on factors other than investment. It involves an assumption that duopoly profits are high relative to the costs of research, which we introduce in section 3.

Section 6 contains normative analysis about the impact of R&D policy on the investment and sharing decisions of firms. For this purpose, we let TS^M and TS^D denote the flow of total surplus in the product market under monopoly and duopoly, respectively. We assume that $TS^M < TS^D$.

2.3 Equilibrium, Payoffs and Sharing Dynamics

Research Histories and Markov States To represent the progress made by the firms, we define a set of research histories. We use the notation (h_1, h_2) where h_i stands for the number of steps that firm i has completed. When firm i completes a research step, h_i increases by one. We refer to research histories where $h_1 = h_2$ as symmetric histories and to those where $h_1 \neq h_2$ as asymmetric histories. At asymmetric research histories, the firms have the opportunity to share research, as described below.

We also define a set of Markov states for the game. The research histories (h_1, h_2) are all Markov states. Below, we describe the available actions at each state and how transitions between states occur.

We first describe actions at the symmetric research histories. At histories (h, h) with $h = 0, 1$, the firms simultaneously decide whether to invest in the next step of research. If both firms invest, they each incur the flow cost c. When one of them, say firm 1, is successful, the state transitions to $(h+1, h)$. At (h, h), if one of the firms, say firm 1, does not invest, the state transitions to (X, h) where X denotes that a firm has exited the game. If both firms drop out at (h, h), the state transitions to (X, X) and the game is over. If one or both firm

[15] As an example, consider a demand function of the type $q_i = (a(1-\gamma) - p_i + \gamma p_j) / (1-\gamma^2)$, where $0 < \gamma < 1$ so that the products are substitutes. The goods are less differentiated the higher is γ. It is possible to show that $\pi^M \leq 2\pi^D$ if γ is sufficiently small. Singh and Vives (1984) show how these demand functions derive from particular consumer preferences. The Hotelling model provides another example of a differentiated duopoly.

invests, then they continue to invest until one of them is successful. At the symmetric history $(2, 2)$, the firms earn duopoly profits.

At asymmetric research histories (h_1, h_2), before deciding whether or not to invest, the firms first decide whether to enter into a licensing agreement. This is a joint decision, not a strategic one. The firms agree to share if and only if doing so increases their joint continuation profits. Although this is not essential to our results, we assume that the leading firm claims the whole surplus from the agreement through a licensing fee paid by the lagging firm. Under this assumption, the leading firm has all of the bargaining power and makes a take-it-or-leave-it licensing fee offer to the lagging firm. If an agreement is concluded, the license fee is paid, the research is shared, and the history transitions to the research history $(h_1 + 1, h_2)$ or $(h_1, h_2 + 1)$ depending on which firm is the leader. If an agreement is not concluded and neither firm is done with research, the firms simultaneously decide whether to invest in the next step of research. If one of the firms is done with research, that firm earns monopoly profits while the other firm decides whether or not to invest.

At the asymmetric states (X, h) and (h, X), there is only one firm in the game. If $h = 2$, the firm is a monopolist. If $h < 2$, the firm decides whether or not to invest.

Markov Perfect Equilibrium A Markov equilibrium consists of sharing decisions and investment strategies. We define two value functions, $V_i(h_1, h_2)$ and $U_i(h_1, h_2)$, which are the expected continuation profits for firm i before and after the sharing decision, respectively. The function $U_i(h_1, h_2)$ is defined only at the histories (h_1, h_2) where a sharing decision is made. Joint profits are denoted with $V_J = V_1 + V_2$ and $U_J = U_1 + U_2$. A Markov strategy for firm i specifies an investment decision at each (h_1, h_2) where an investment decision is made.

Definition 1 *A pure strategy Markov Perfect Equilibrium (MPE) consists of Markov strategies for* $i = 1, 2$ *and value functions* $V_i(h_1, h_2)$ *and* $U_i(h_1, h_2)$ *such that (i) the sharing decision maximizes the joint continuation profits at each asymmetric history* (h_1, h_2) *and (ii) the investment decision for firm* i *maximizes the individual continuation profits of firm* i *given the strategy of firm* j *at each* (h_1, h_2) *where an investment decision is made.*

The sharing decisions in (i) are done through a licensing process that maximizes the joint profits $V_J(h_1, h_2)$. When firm 1 is the leader, the joint profits are $V_J(h_1, h_2+1)$ if the firms share and $U_J(h_1, h_2)$ if they do not, so that $V_J(h_1, h_2)$ is the larger of these two payoffs. The following sharing condition formalizes (i) and describes whether the firms enter into a licensing agreement:

$$V_J(h_1+1, h_2) > U_J(h_1, h_2). \quad (1)$$

If (1) does not hold, then $V_i(h_1, h_2) = U_i(h_1, h_2)$. If (1) holds, the licensing fee affects the value function $V_i(h_1, h_2)$.[16] When firm 1 is the leader and the firms share, the value functions V_i are given by

$$\begin{aligned} V_1(h_1, h_2) &= F(h_1, h_2) + V_1(h_1, h_2+1) \\ V_2(h_1, h_2) &= V_2(h_1, h_2+1) - F(h_1, h_2). \end{aligned} \quad (2)$$

Because the leader is assumed to claim the entire surplus from the agreement, the licensing fee leaves the lagging firm just indifferent between accepting and rejecting the agreement so that $V_2(h_1, h_2) = U_2(h_1, h_2)$. This implies that the licensing fee is

$$F(h_1, h_2) = V_2(h_1, h_2+1) - U_2(h_1, h_2). \quad (3)$$

The investment decisions in (ii) are determined by Bellman equations that characterize the optimal decisions for each firm. These equations are included in section A of the appendix. Here, we illustrate with an example. At $(1, 0)$, after a decision not to share, both firms make investment decisions. If firm 2 invests, firm 1's value function is given by:

$$U_1(1,0) = \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(2,0) + \alpha V_1(1,1) - c\right) dt\right\}.$$

The integral expression is the expected payoff to firm 1 if it invests. If firm 1 invests, then it incurs the flow cost c. In each instant of time, there is a probability α that firm 1 is successful and the state transitions to $(2, 0)$. Similarly, there is a probability α that firm 2 is successful and the state transitions to $(1, 1)$.

[16] Since the licensing fee is a transfer between the firms, it does not affect their joint payoffs or the sharing decision.

Sharing Dynamics Our analysis focuses on how the incentives to license research change over time. For ease of exposition, we restrict attention to symmetric MPE and consider sharing histories such that firm 1 is the leader. To analyze the dynamics of sharing, we compare the sharing conditions of the firms at $(2,1)$ and $(1,0)$. At each of these histories, the leader is exactly one step ahead of the lagging firm. We say that the sharing incentives are decreasing over time if the firms have a stronger incentive to share at $(1,0)$ than at $(2,1)$. Conversely, sharing incentives are increasing over time if the firms have a stronger incentive to share at $(2,1)$ than at $(1,0)$.

The firms also decide whether to share at $(2,0)$. Because the number of steps that the lagging firm is behind is a factor in the firms' sharing conditions, we do not analyze dynamics for this state. In a game with more than two periods, dynamics across histories where one firm is two steps behind could be analyzed.

In our discussion, we refer to sharing patterns. A sharing pattern simply describes the sharing decisions at $(1,0)$ and $(2,1)$. The possible sharing patterns are (S,S), (S,NS), (NS,NS), and (NS,S), where S stands for sharing and NS stands for no sharing. When sharing incentives are decreasing over time, the pattern (NS,S) does not arise. Conversely, when sharing incentives are increasing over time, the pattern (S,NS) does not arise.

2.4 Joint Profit Maximization Benchmarks

We discuss two benchmarks. First, we consider what the firms would do if they could make all of their decisions (investment, sharing and production) jointly. We call this the *joint profit maximization benchmark*. In this benchmark, it is optimal for the firms to cooperate in the product market and earn flow profits of π^J. During the research process, it is always optimal for the firms to share research successes as soon as one of them is ahead. There are two reasons for this. The first is that duplication of research is purely wasteful. We refer to this as the *duplication effect*. In our analysis, this benefit is captured by a savings of the flow costs c of research. It is present in the analysis of every sharing decision. The second reason is that sharing can allow the firms to reach the product market sooner. We refer to this as the *speed effect*. The speed effect is present only when neither firm has completed both steps of research, so at $(1,0)$ and $(0,1)$. By sharing at $(1,0)$, the firms can both work on the second step of research, thereby speeding up the time until one of them completes the second step. When the firms make production decisions jointly, there is no downside to sharing to counterbalance these positive effects and it is always optimal for the firms to share.

In the joint profit maximization benchmark, the firms make investment decisions at $(0,0)$ and $(1,1)$. They invest provided the expected payoffs are positive, and if one firm invests, so does the other. The reason for this is that in the Poisson discovery process with identical firms, if it is optimal for one firm to invest in a step, then it is optimal for both to invest even if the firms could agree to have just one of them to invest. This speeds up the time to innovation, and the benefits of the time savings outweigh the costs of running simultaneous facilities.

In our model, firms are not allowed to make joint production decisions in the product market. This motivates a second benchmark, which we call the *constrained joint profit maximization benchmark.* The firms again make all of their investment and sharing decisions jointly, but they do not make joint production decisions. The best outcome they can achieve is flow profits of $2\pi^D$ or π^M, whichever is higher. When $\pi^M > 2\pi^D$, there is a downside to sharing which we refer to as the *rivalry effect.* Sharing can reduce the joint profits of the firms if it enables a lagging firm to enter the product market and disrupt the flow of monopoly profits to the leading firm. In this case, when one firm completes both steps of research, it is optimal for the lagging firm to exit the race. This allows the firms to achieve the flow profits of π^M. When $\pi^M \leq 2\pi^D$, the firms would like to achieve $2\pi^D$ as the flow profits in the product market and they can do this by sharing once one firm completes both steps of research. Since either the lagging firm exits or the firms share after one firm finishes both steps of research, there is no downside to sharing the first step at $(1,0)$ and $(0,1)$, so the firms share at these histories. At $(0,0)$ and $(1,1)$, the firms invest if their expected payoffs are positive. For the reasons stated above, if it is optimal for one firm to invest, then it is optimal for both firms to invest.

3 Optimal Sharing Dynamics

In this section, we analyze the dynamics of sharing between two competing firms. The incentives to share depend on joint profits, not individual profits. We first develop our main result on the dynamics of sharing incentives, and then discuss the dynamics of the individual profits and licensing fees. For simplicity, we assume that the firms never drop out of the R&D race. This allows us to focus on the dynamics of sharing that are driven by factors other than investment. In section 4, we briefly discuss some results that arise when this assumption is relaxed.

To identify the parameter values such that the firms always invest, we consider the continuation payoff that a firm would receive by conducting two steps of research on its own and then earning duopoly profits in the output market. Intuitively, this is the worst possible position for a firm. Because a firm can achieve this payoff without sharing, it is a lower bound on the firm's payoff from investing at any history and in any equilibrium.

We compute this payoff by working backwards. After completing the two steps of research, the firm produces output as a duopolist to earn $\widetilde{\pi}^D = \frac{\pi^D}{r}$. To complete the second step of research, the firm invests a flow cost of c. The firm's expected payoff is

$$\int_0^\infty e^{-(\alpha+r)t} \left(\alpha\widetilde{\pi}^D - c\right) dt = \frac{\alpha\widetilde{\pi}^D - c}{\alpha + r}. \tag{4}$$

The firm invests in the second step if $\pi^D > \frac{cr}{\alpha}$. To complete the first step of research, the firm again invests a flow cost of c and the hazard rate is again α. The firm's expected payoff is

$$\int_0^\infty e^{-(\alpha+r)t} \left(\alpha\left(\frac{\alpha\widetilde{\pi}^D - c}{\alpha + r}\right) - c\right) dt = \frac{\alpha\left(\frac{\alpha\widetilde{\pi}^D - c}{\alpha+r}\right) - c}{\alpha + r}. \tag{5}$$

The firm invests in the first step if $\pi^D > \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right)$.

In any equilibrium of the dynamic sharing game, the payoffs (4) and (5) are the payoffs $U_2(2,1)$ and $U_2(2,0)$ to the lagging firm from investing at $(2,1)$ and $(2,0)$, respectively, after a decision not to share. At $(2,0)$, this is so regardless of whether or not the firms share at $(2,1)$ because a lagging firm does not have any bargaining power in the determination of the licensing fee at $(2,1)$. We make the following assumption.

Assumption 1 $\pi^D > \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right)$.

Under Assumption 1, in every Markov Perfect Equilibrium of the game, firms do not exit at any history either on or off the equilibrium path. Assumption 1 implies that environments without exit arise when competition in the product market is relatively soft so that duopoly profits are high by comparison to the costs of research (in terms of time and money).

To explore the dynamics of sharing, we compare the firms' incentives to share early stage research at the history $(1,0)$ to their incentives to share late stage research at the history $(2,1)$. Because each of the research steps in our model is identical from a technology standpoint, a conclusion that sharing incentives must change over time is not obvious. However, the firms are closer to the product market at $(2,1)$ than they are at $(1,0)$. A basic intuition is that as firms approach the end of the research process, their decisions might increasingly reflect the impending rivalry. If so, then firms might be less likely to share late stage research. Proposition 1 records the dynamic equilibrium sharing patterns that arise when firms do not exit the game. We discuss below how these equilibria express this basic intuition.

Proposition 1 *Under Assumption 1, we have:*

(i) When $2\pi^D < \frac{(2\alpha^2 - r^2)\pi^M - (2\alpha + r)^2 c}{(3\alpha^2 + 2\alpha r)}$*, there is a unique MPE where the firms do not share at any history.*

(ii) When $\frac{(2\alpha^2 - r^2)\pi^M - (2\alpha + r)^2 c}{(3\alpha^2 + 2\alpha r)} < 2\pi^D < \pi^M - c$*, there is a unique MPE where the firms share at* $(1,0)$ *but not at* $(2,0)$ *or* $(2,1)$*.*

(iii) When $\pi^M - c < 2\pi^D$*, there is a unique MPE where the firms share at every history.*

The proposition is proved in section B of the appendix.[17] A first observation about Proposition 1 is that when there is less rivalry between the firms, they share more often. The three parts of Proposition 1 correspond to different levels of competition, as represented by $2\pi^D$. In (iii), competition is weak so that duopoly profits are close to monopoly profits. Here, the firms share both steps of research. As $2\pi^D$ decreases so that we move from (iii) to (ii), sharing breaks

[17] On the boundaries of the regions (i),(ii), and (iii), the firms are indifferent between sharing and not sharing or between investing and not investing at one or more histories. As a result, there are multiple equilibria that are payoff equivalent.

down at $(2,1)$. As $2\pi^D$ decreases further so that we move from (ii) to (i), sharing breaks down at $(1,0)$ as well. Hence, when sharing breaks down, it breaks down at the later history first. This comparative static reflects the fact that firms' incentives to share at $(1,0)$ are stronger than at $(2,1)$.

A second observation is that the sharing pattern (NS,S) does not arise in equilibrium. We would expect to see only the sharing patterns (S,S), (S,NS) and (NS,NS). Sharing may break down as the firms approach the product market, as in (ii) where the firms share at $(1,0)$ but not at $(2,1)$. However, the reverse dynamic is not possible. This also reflects the fact that firms' incentives to share at $(1,0)$ are stronger than at $(2,1)$.

We now explain the underlying dynamics behind Proposition 1 by comparing the sharing conditions at $(2,1)$ and $(1,0)$. As mentioned in section 2.4, the sharing decision depends on the balance of three effects: the rivalry effect, the duplication effect and the speed effect. Consider first the sharing condition at $(2,1)$. Firms share at $(2,1)$ if $V_J(2,2) > U_J(2,1)$. If the firms share, they compete as duopolists in the product market. Their continuation profits are $V_J(2,2) = 2\widetilde{\pi}^D$. If the firms do not share, the leading firm earns a flow profit of π^M and the lagging firm invests c until the lagging firm finishes. Their joint continuation profits are $U_J(2,1) = \int_0^\infty e^{-(\alpha+r)t}\left(\pi^M - c + \alpha V_J(2,2)\right)dt = \frac{\pi^M + 2\alpha\widetilde{\pi}^D - c}{\alpha+r}$. The sharing condition simplifies to

$$2\pi^D - \pi^M + c > 0. \qquad (6)$$

The term $2\pi^D - \pi^M$ in condition (6) captures the rivalry effect. This term must be negative for there to be a downside to sharing. The term c is the duplication effect.[18]

We show that the sharing condition at $(1,0)$ is weaker than (6). There are two explanations depending on whether (6) holds or fails. Consider first the case when condition (6) fails, as in (i) and (ii) in Proposition 1. As shown in the appendix, the sharing condition at $(2,0)$ is also given by condition (6). Thus, the firms do not share at $(2,0)$ either. At the earlier history $(1,0)$, the sharing condition is $V_J(1,1) > U_J(1,0)$, which simplifies to condition (18) in the

[18] For an alternative interpretation, condition (6) can also be written as $\pi^M - \pi^D < c + \pi^D$. The LHS represents the per-period loss of the leader due to sharing while the RHS represents the per-period gain of the lagging firm due to sharing.

appendix and is the boundary between regions (i) and (ii). At $(1, 0)$, there is a new benefit of sharing that did not exist at $(2, 1)$. The lagging firm now has a chance of finishing first. If the firms knew that firm 2 would finish first, they would want to share at $(1, 0)$ so as to realize monopoly profits sooner. This is the speed effect. In contrast, if the firms knew that firm 1 would finish first, then they would not want to share at $(1, 0)$ because this shortens the duration of monopoly profits. We can re-write the sharing condition (18) in the following way:

$$\beta(\pi^M + c) + (1 - \beta)(2\pi^D - \pi^M + c) > 0, \tag{7}$$

where $\beta = \frac{\left(1+\frac{r}{\alpha}\right)^2}{\left(2+\frac{r}{\alpha}\right)^2}$. The second term in (7) is the net loss in joint flow profits when the leading firm finishes first. This is the same as condition (6) and is negative. The first term in (7) is the increase in joint flow profits when the lagging firm finishes first. Here, the firms jointly benefit from replacing the lagging firm's R&D costs $-c$ with monopoly profits π^M. The net benefit, $\pi^M + c$, which is due both to the speed effect (monopoly profits π^M are earned sooner) and the duplication effect (flow costs c are saved), is positive. Since $\beta > 0$, condition (7) is easier to satisfy than (6), and hence sharing incentives are decreasing.

The β and $(1 - \beta)$ can be interpreted as weighted probabilities. There is a weighted probability β that the lagging firm finishes first and a weighted probability $(1 - \beta)$ that the leading firm finishes first. At $(2, 1)$, $\beta = 0$ because the leading firm is already done. The duplication effect c has the same weight in both sharing conditions (7) and (6) in so far as the coefficient on c is simply 1. In this sense, it is the changing importance of the rivalry and speed effects that drives the sharing dynamics, not the duplication effect. When β is larger, there is more weight on the speed effect, so there is more incentive to share. The magnitude of β depends on how impatient the firms are. The ratio $\frac{r}{\alpha}$ in the expression for β can be interpreted as a discount factor. The underlying interest rate r is adjusted by the effectiveness α of the research technology. The probability β is increasing in $\frac{r}{\alpha}$ so that when the firms are more impatient, the incentives to share are stronger.

When condition (7) holds, there is a unique MPE with the sharing pattern (S,NS). When it fails, there is a unique MPE with the sharing pattern (NS,NS).

Consider next the case when condition (6) holds, as in (iii) in Proposition 1. Because (6) holds, the firms share at $(2,1)$. As shown in the appendix, the sharing condition at $(2,0)$ is again given by condition (6), so the firms share at $(2,0)$. The sharing condition at $(1,0)$ simplifies to

$$\pi^D + c > 0. \tag{8}$$

This holds trivially so that the equilibrium sharing pattern is (S,S). This result is explained by dynamic feedback effects. Since the firms share at both $(2,1)$ and $(2,0)$, neither firm can ever earn monopoly profits and, thus, there is no rivalry effect earlier in the game at $(1,0)$. Sharing merely reduces the expected time to market (the speed effect) and expected R&D costs (the duplication effect) by enabling the lagging firm to finish sooner. The sharing condition captures the change in joint flow profits when this happens.

In summary, there are two explanations for why sharing patterns are decreasing over time. The first explanation is that if the firms do not share at $(2,1)$, sharing at $(1,0)$ may still be beneficial because it reduces the time to market by enabling the lagging firm to finish first. This speed effect is not present at the histories $(2,0)$ and $(2,1)$ where one firm has already reached the market. The second explanation is that if the firms share at $(2,1)$ and $(2,0)$, this eliminates the only cost of sharing which is the rivalry effect. As a result, at $(1,0)$, neither firm expects to earn monopoly profits in the future and the sharing condition at $(1,0)$ holds trivially. It is interesting to note that the dynamics described above continue to hold when research costs c are zero. Hence, savings of duplicated R&D costs are not the only reason the firms find it optimal to share. Firms are also motivated to share by the speed effect.[19]

We conclude by briefly discussing individual payoffs and licensing fees. Since sharing decisions are made jointly, they do not depend on this analysis. However, it is still interesting to consider whether the licensing fees have the same dynamics as the sharing incentives. In section C of the appendix, we consider the MPE in (iii) of Proposition 1 in which the firms share at all

[19] These results extend to a model with three research steps. A proof is available on request. With a three-step research process, we can compare histories where the leader is one step ahead of the lagging firm (i.e., $(1,0)$, $(2,1)$, and $(3,2)$), and histories where the leader is two steps ahead of the lagging firm (i.e., $(2,0)$ and $(3,1)$). The sharing conditions all have the form (7) at all histories and in every equilibrium, where $\beta \in [0,1)$ depends on the history and the future sharing decisions. In each equilibrium, when we compare the sharing conditions at two histories with the same gap, we find that the value of β is higher at the earlier history. This means that the speed effect is more important relative to the rivalry effect, and it gives us the result that sharing incentives are decreasing.

histories. We find that both firms have a higher payoff at $(2,1)$ than at $(1,0)$. Essentially, this is because costs are invested upfront while profits are earned later and are discounted. Hence, as the game progresses, individual payoffs rise. This is in contrast with the sharing incentives which decrease over time. The dynamics of the licensing fees depend on the magnitude of the discount factor $\frac{r}{\alpha}$. When $\frac{r}{\alpha}$ is sufficiently high, the payoffs increase significantly over time, and the licensing fees increase along with them so that $F(2,1) > F(1,0)$. When $\frac{r}{\alpha}$ is low, however, the licensing fees have the same dynamics as the sharing incentives so that $F(2,1) < F(1,0)$.

4 Optimal Sharing Dynamics and Investment

In this section, we briefly discuss what happens when Assumption 1 does not hold. Erkal and Minehart (2012) provides a more complete analysis. If duopoly profits are too low to keep the lagging firm in the race at all histories, the firms can prevent the erosion of monopoly profits (the rivalry effect) by causing the lagging firm to exit the race. The dynamics of sharing are now also shaped by whether a sharing decision can induce exit.

The most important new issue for the sharing dynamics is that a lagging firm is more likely to drop out when it has more research left to complete. Given this, the firms may be less likely to share earlier in the game. In Erkal and Minehart (2012), we demonstrate an equilibrium in which the increasing sharing pattern (NS,S) arises on the equilibrium path. In the equilibrium, duopoly profits are high enough so that the lagging firm stays in the race at $(1,0)$ and $(2,1)$ after a decision not to share. $(2,0)$ is the only history at which the lagging firm drops out after a decision not to share (which implies, from (5), $\pi^D < \frac{cr}{\alpha}\left(2 + \frac{r}{\alpha}\right)$). Because of the rivalry effect, the firms have a strong incentive to forego sharing at $(1,0)$ in order to reach $(2,0)$. At $(2,0)$, the firms do not share, the lagging firm drops out, and the leading firm then earns monopoly profits forever. The sharing pattern (NS,S) arises on the equilibrium path when, after choosing not to share at $(1,0)$, the firms next reach the history $(1,1)$ rather than $(2,0)$. The game then proceeds to $(2,1)$ or $(1,2)$, at which point the firms share step 2.

Hence, exit by the lagging firm at $(2,0)$ may weaken the sharing incentives earlier in the game. If, in addition to $(2,0)$, the lagging firm also exits at $(2,1)$ after a decision not to share (which happens, from (4), if $\pi^D < \frac{cr}{\alpha}$), sharing incentives earlier in the game are enhanced. We show that if the firms rivalrous (i.e., if $\pi^M > 2\pi^D$), the firms choose NS at $(2,1)$ and the lagging firm then drops out. Because the firms will never compete as duopolists, there is no rivalry effect at $(1,0)$ and $(2,0)$. The sharing conditions at these earlier histories hold trivially, and the equilibrium sharing pattern is (S,NS). Recall from section 3 that when the firms share at $(2,1)$ and $(2,0)$, the rivalry effect is eliminated at $(1,0)$ because there is no way to achieve monopoly profits. Here, the rivalry effect is again eliminated at $(1,0)$, but now it is because monopoly profits are assured. The firms are willing to exploit the efficiencies of sharing at $(1,0)$ because they know that one of them will exit the race later. Along the equilibrium path, the firms achieve the constrained joint profit maximization benchmark that was introduced in section 2.4.

5 Asymmetric Firms

So far we have assumed that firms are symmetric in their research capabilities to focus on the impact of uncertainty and progress on the firms' sharing decisions. In this section, we analyze to what extent our conclusions in section 3 apply when firms have different abilities to conduct different stages of research. For example, in the biotechnology industry, alliances often involve a firm which has developed expertise in research on a particular biotechnology and a large pharmaceutical which may be better able to bring the product through the clinical testing and regulatory approval process to the market (Lerner and Merges, 1998).

We consider an environment where one of the firms is better at one step of research than the other step of research. Assume that firm 1 has a cost of c_1 for both steps of research, but firm 2 has a cost of c_2^1 for the first step and c_2^2 for the second step. This is the simplest way to model two firms with different relative advantages. With this modification in the model, Assumption 1 becomes Assumption 1$'$:

Assumption 1$'$ $\pi^D > \max\left\{\frac{c_1 r}{\alpha}\left(2+\frac{r}{\alpha}\right), \frac{c_2^1 r}{\alpha}\left(1+\frac{r}{\alpha}\right)+\frac{c_2^2 r}{\alpha}\right\}$.

As before, the assumption implies that in every Markov Perfect Equilibrium of the game, firms do not exit at any history either on or off the equilibrium path. The two terms inside the curly brackets are the continuation payoffs that firm 1 and firm 2 would receive by conducting two steps of research on their own and then earning duopoly profits in the output market, respectively.

Due to the cost asymmetry, the dynamics of sharing incentives depend on which firm is the leader. We have the following result.

Proposition 2 *Suppose Assumption 1′ holds. Then:*

(i) Consider the histories $(0,1)$ *and* $(1,2)$ *where firm* 2 *is the leader. If the firms share at* $(1,2)$ *in an MPE, they also share at* $(0,1)$*.*

(ii) Consider the histories $(1,0)$ *and* $(2,1)$ *where firm* 1 *is the leader. If* $c_2^1 \geqslant c_2^2$*, then if the firms share at* $(2,1)$ *in an MPE, they also share at* $(1,0)$*. However, for some values of* $c_2^1 < c_2^2$*, there exists an MPE such that the firms share at* $(2,1)$*, but they do not share at* $(1,0)$*.*

The novel result in Proposition 2 is that the sharing pattern (NS,S) may arise when firm 1 is the leader. This means if the firms share at $(2,1)$, they do not necessarily share at $(1,0)$. To explore this, we derive an equilibrium in section D of the appendix where the firms share at the four histories $(2,1)$, $(1,2)$, $(2,0)$ and $(0,2)$. As in the symmetric model, this future sharing eliminates the rivalry effect at $(1,0)$. However, the firms may still decide not to share. From (22) in the appendix, the sharing condition at $(1,0)$ is

$$\beta\left(2\pi^D + c_1 + c_2^2\right) + (1-\beta)(c_2^1 - c_2^2) > 0, \tag{9}$$

where $\beta = \frac{\alpha}{(3\alpha+r)}$. The new term $c_2^1 - c_2^2$ captures the change in investment costs when the lagging firm stops research on step 1 and begins research on step 2. We refer to this as the *progress effect.* If $c_2^1 - c_2^2 < 0$, this is a loss and (9) does not always hold. When (9) fails, the firms share at $(2,1)$ but not at $(1,0)$. By not sharing at $(1,0)$, the firms prevent firm 2 from starting to work on step 2, where it would incur higher research costs. If firm 1 subsequently completes step 2, firm 2 will never have to work on it.[20]

The first term in (9) captures the speed effect (because the firms earn duopoly profits $2\pi^D$ sooner) and the duplication effect (because the firms save research costs $c_1 + c_2^2$ by finishing the

[20] The firms would attain even higher joint profits if firm 2 were simply to refrain from conducting further research at $(1,0)$. However, by assumption, the firms cannot agree to this.

project sooner). These benefits are both realized at the point that the firms enter the product market. By contrast, the progress effect is realized immediately when the firms share at $(1, 0)$. Hence, when firms are more impatient (i.e., when the discount factor $\frac{r}{\alpha}$ is high so that β is low), they put more weight on the progress effect and may decide against sharing.

Proposition 2 shows that when research costs increase over time, the firms may have a stronger incentive to share as the race progresses. However, if research costs decrease or do not change over time, the sharing incentives continue to be decreasing. Proposition 2 implies that it is the asymmetry of costs for a single firm that changes the sharing dynamics, not the asymmetry between the firms.

6 Patent Policy

In the analysis so far, we have assumed that firms face the same research cost c regardless of whether the step they are working on has been patented. In this section, we assume patenting increases the research cost faced by a lagging firm by forcing it to invent around (Gallini, 1992). Thus, patent policy may change the sharing dynamics by affecting the magnitude of the duplication effect.

We investigate how patent policy can be used to change the sharing incentives of firms to increase social welfare. We consider a social planner maximizing expected social welfare at $(0, 0)$, defined as the expected value of the flow of total surplus in the product market net of the firms' expected flow costs of research in the R&D phase. Recall from section 2.2 that the flow total surplus under duopoly and monopoly are TS^D and TS^M, respectively, where $TS^D > TS^M$. The expected social welfare $W(h_1, h_2)$ at a history is derived in the same way as the value functions for the firms.[21]

We structure our discussion around two policy changes. The first policy provides broader patent protection for early research outcomes. The second policy provides broader patent

[21] We define social value functions $W(h_1, h_2)$ and $Z(h_1, h_2)$ as the social welfare before and after a sharing decision is made at (h_1, h_2), respectively. To illustrate how W and Z are derived, consider the game in section 3. If the firms share at $(2, 1)$ and $(1, 2)$, then $W(2, 1) = W(1, 2) = \int_0^\infty e^{-rt} TS^D dt = \frac{1}{r} TS^D = \widetilde{TS}^D$. Working backwards, at the history $(1, 1)$, $W(1, 1) = \int_0^\infty e^{-(2\alpha+r)t}(\alpha W(2, 1) + \alpha W(1, 2) - 2c)dt = 2(\alpha\widetilde{TS}^D - c)/(2\alpha + r)$.

protection for late research outcomes. Specifically, let $c_p^1 \geq c$ and $c_p^2 \geq c$ stand for the research cost of the lagging firm for the first and second research steps, respectively. The gap $c_p^i - c$ is the extra cost of inventing around the leading firm's patent, and is a measure of the breadth of the patent policy. An early stage policy c_p^1 changes the flow cost of research for the lagging firm at the histories $(1,0)$ and $(2,0)$. A late stage policy c_p^2 changes the flow cost of research for the lagging firm at the history $(2,1)$.

With this modification in the model, Assumption 1 becomes Assumption 1$''$:

Assumption 1$''$ $\pi^D > \frac{r}{\alpha}\left(1+\frac{r}{\alpha}\right)c_p^1 + c_p^2$.

When this condition holds, because investment incentives are adequate, we can focus on the impact patent policy has on sharing decisions.[22] From the social planner's perspective, sharing is always desirable. Sharing increases social welfare by eliminating duplication and speeding up the time for the first firm to reach the product market. Sharing also helps the lagging firm to reach the product market sooner. Since total surplus is higher under duopoly than monopoly, the social planner also values this effect.

Intuitively, one would expect broader patent policy to encourage firms to share by increasing the cost to the lagging firm of working around the leading firm's patent. If this is the case, then broader patent protection increases social welfare. We find that this intuition holds for broader patent protection of early research outcomes, but, surprisingly, it does not necessarily hold for broader patent protection of late research outcomes.

Our first result considers broader patent protection of early research outcomes. We focus on environments where the firms do not share at any history when $c_p^1 = c_p^2 = c$. We consider how the MPE changes as c_p^1 is increased so that $c_p^1 > c_p^2 = c$. Proposition 3 confirms the basic intuition that broad patent policy can enhance social welfare by encouraging firms to share their research.

[22] When the expression holds, it is profitable for the lagging firm to invest at $(2,0)$ despite having to pay c_p^1 to research the first step and c_p^2 to research the second step. The expression shows that increases in the costs of first-step vs. second-step research will affect investment incentives differently. Specifically, increases in the patent protection c_p^1 of early research reduces the continuation profits of the lagging firm at $(2,0)$ more than do equivalent increases in c_p^2. The expression holds more easily as $\frac{r}{\alpha} \to 0$.

Proposition 3 *(Early Stage Patent Policy) Consider an industry with parameters* π^M, π^D, r, α, c, $c_p^1 \geq c$ *and* $c_p^2 = c$ *such that Assumption 1″ holds. Suppose that when* $c_p^1 = c$*, the equilibrium research outcome does not involve sharing at any of the research histories. There exist threshold levels* $\bar{\bar{c}}_p^1 > \bar{c}_p^1 > c$ *such that:*

(i) when $\bar{c}_p^1 > c_p^1 > c$*, the firms do not share at any history;*

(ii) when $\bar{\bar{c}}_p^1 > c_p^1 > \bar{c}_p^1$*, the firms share at* $(1,0)$*, but not at* $(2,0)$ *or* $(2,1)$*;*

(iii) when $c_p^1 > \bar{\bar{c}}_p^1$*, the firms share at* $(1,0)$ *and* $(2,0)$*, but not at* $(2,1)$*.*

Notice that in Proposition 3, broader patent protection of early research has no impact on the decision to share the second step. This is because first-stage research costs are irrelevant to the sharing decision at $(2,1)$. Notice also that in Proposition 3, as c_p^1 increases, the firms start to share at $(1,0)$ before they start to share at $(2,0)$. At both histories, the firms are deciding whether to share the first research step as it is subjected to broader patent protection. The difference is that at $(2,0)$, the leader earns monopoly profits while at $(1,0)$, the leader is engaged in research, and there is still uncertainty about which firm will finish first. The fact that the leading firm is making monopoly profits at $(2,0)$ makes the rivalry effect very strong, and the firms benefit by delaying the lagging firm's entry into the market.

Patent policy increases social welfare $W(0,0)$ in parts (ii) and (iii) of Proposition 3. This is because sharing at $(1,0)$ eliminates duplication of research and speeds the time to market. In part (i), the patent policy reduces social welfare. Here, the policy is not strong enough to induce sharing and acts only to increase the lagging firm's research costs at $(1,0)$ and $(2,0)$.

In the next proposition, we consider the impact of broader patent protection of late research. For this, we assume $c_p^2 > c_p^1 = c$ and analyze what happens as c_p^2 increases. Although policies that affect costs of early research do not have an impact on the incentives to share late research outcomes, the converse is not true.

To illustrate, we consider a region of parameters such that when $c_p^2 = c$, the equilibrium research outcome involves sharing at $(1,0)$ but no sharing at $(2,0)$ or $(2,1)$.

Proposition 4 *(Late Stage Patent Policy) Consider an industry with parameters* π^M, π^D, r, α, c, $c_p^1 = c$ *and* $c_p^2 \geq c$ *such that Assumption 1″ holds. Suppose that when* $c_p^2 = c$*, the* *equilibrium research outcome involves sharing at* $(1, 0)$ *but no sharing at* $(2, 0)$ *and* $(2, 1)$*. In addition, assume that* $c < \overline{c} = (\pi^M - 2\pi^D) - \left(\frac{2\alpha+2r}{4\alpha+3r}\right)(\pi^M - \pi^D)$*. There exist threshold levels* $\overline{c} > \overline{\overline{c}}_p^2 > \overline{c}_p^2 > c$ *such that:*

(i) when $\overline{c}_p^2 > c_p^2 > c$*, the firms share at* $(1, 0)$ *but not at* $(2, 0)$ *or* $(2, 1)$*;*

(ii) when $\overline{\overline{c}}_p^2 > c_p^2 > \overline{c}_p^2$*, the firms do not share at any history;*

(iii) when $\overline{c} > c_p^2 > \overline{\overline{c}}_p^2$*, the firms share at* $(2, 1)$ *but not at* $(2, 0)$ *or* $(1, 0)$*.*

In Proposition 4, the policy targets sharing of late research and such sharing is achieved, but at the cost of discouraging sharing at $(1, 0)$. As patent protection of late research broadens, it becomes more expensive for firm 2 to conduct research at $(2, 1)$. This introduces a progress effect similar to the one we discussed in section 5. The progress effect discourages sharing at earlier histories because a decision not to share step 1 delays the lagging firm from reaching step 2 and, hence, delays the research expense that the lagging firm incurs at step 2. This explains why sharing breaks down at $(1, 0)$ in part (ii) of Proposition 4. In part (iii), c_p^2 is so high that the duplication effect leads the firms to share at $(2, 1)$. As a result, c_p^2 no longer impacts the sharing incentives at earlier histories. With the rivalry and progress effects eliminated at $(2, 1)$, it might be reasonable to expect that the firms would share at $(1, 0)$. This does not happen, however, because the rivalry effect is still present at $(2, 0)$.[23] The sharing condition at $(2, 0)$ is $\pi^M - 2\pi^D < c$, which fails to hold for the selected parameters so that the firms do not share. This decision then feeds back to deter sharing at $(1, 0)$. Because the firms will share at $(2, 1)$, the only way firm 1 can expect to earn π^M is by reaching $(2, 0)$, and this can only be achieved if the firms do not share at $(1, 0)$.

Social welfare is reduced in parts (i) and (ii) of Proposition 4 as patent protection c_p^2 increases. In part (i), this is because the lagging firm incurs higher costs to invent around the patent at $(2, 1)$ while the sharing and investment decisions of the firms are unchanged. In part (ii), social welfare is further reduced when sharing breaks down at $(1, 0)$. In part (iii), as c_p^2 increases further, the firms begin to share at $(2, 1)$. This increases social welfare by converting

[23] In section 3.1, when the rivalry effect was eliminated at $(2, 1)$, it was also eliminated at $(2, 0)$. This does not happen here because the patent policy introduces an asymmetry in the costs for the two steps of research.

monopoly profits to duopoly profits. It also eliminates the costly duplication of second stage research by the lagging firm, which enhances social welfare.

A natural question is whether welfare is higher under the patent policy in part (iii) than in the basic model where $c_p^1 = c_p^2 = c$. In part (iii), the firms share late stage research at $(2, 1)$, but they do not share early stage research at $(1, 0)$. In the basic model, we have the opposite sharing pattern. The firms share early stage research at $(1, 0)$, but they do not share late stage research at $(2, 1)$. It is not obvious which sharing pattern is preferred by the social planner. Sharing of late stage research encourages competition in the product market, but sharing of early stage research allows the firms to reach the product market earlier (the speed effect). Considering the expected social welfare $W(0, 0)$ under both scenarios, it is straightforward to show that the social planner would prefer sharing of early stage research under a relatively mild condition, given by $TS^M > \frac{1}{2}TS^D$. To see this, ignore the flow costs of R&D for the moment. Sharing at $(2, 1)$ increases the flow of social surplus from TS^M to TS^D so that the change in surplus is $TS^D - TS^M > 0$. By contrast, sharing at $(1, 0)$ reduces the time until the first firm reaches the product market. When this happens, social surplus increases from 0 to TS^M so that the change in surplus is $TS^M > 0$. Hence, if $TS^M > TS^D - TS^M$ the social planner prefers the sharing pattern in which the firms share at $(1, 0)$ to the sharing pattern in which they share at $(2, 1)$. This is also true after taking the flow costs of R&D into account.

Propositions 3 and 4 consider patent policies that target one stage of research only. Suppose instead that the patent policy increases the cost in both stages equally so that $c_p^1 = c_p^2 = c_p > c$. Because the analysis of this scenario is similar to Propositions 3 and 4, we do not include a formal proposition, but some observations are in order. A first observation is that the progress effect identified in Proposition 4 no longer arises because it is equally costly for a lagging firm to work on step 1 and step 2. A second observation is that it is more difficult to use patent policy to induce sharing when the firms are closer to the product market. That is, if we start with an environment where there is no sharing at any of the histories when $c_p = c$, then as c_p increases, we first get sharing at $(1, 0)$. With further increases in c_p, the firms simultaneously switch from not sharing to sharing at both $(2, 0)$ and $(2, 1)$. Hence, a social planner can achieve sharing at all histories, but the policy must involve stronger patent protection than is needed to achieve sharing at $(1, 0)$.

So far, we have considered how patent policy can be used to increase the duplication effect and hence encourage sharing.[24] Duplication can also be avoided by having the lagging firm drop out (as considered in section 4). Patent policy can be used to encourage lagging firms to drop out, as can be seen from the fact that Assumption $1''$ is harder to satisfy when either c_p^1 or c_p^2 is increased. Such policies may not always harm social welfare. They involve a trade-off between avoiding wasteful duplication and reducing competition, and, other things equal, an impatient social planner with a sufficiently high discount factor $\frac{r}{\alpha}$ will prefer the immediate savings of R&D costs to the future harm on competition.[25]

7 Conclusion

A fundamental question in the economics of R&D is how competition affects the incentives for cooperation in R&D. This paper analyzes this question from a dynamic perspective and considers how the incentives to share knowledge change over time as a research project reaches maturity in the context of technological competition. For symmetric firms, sharing dynamics is shown to be driven by three effects: the duplication effect, speed effect and rivalry effect. While the first two effects work in favor of sharing, the last one works against it. Hence, whether the firms share or not depends on how strong the rivalry effect is relative to the other two effects. When the rivalry effect is eliminated, the firms always share.

Our results for symmetric firms reveal that both how close firms are to product market competition and how intense that competition is shape the firms' sharing behavior. We find that when lagging firms do not exit the race, the rivalry effect becomes stronger as the firms approach the product market so that the firms' incentives to share decrease. A related result is that a lower level of duopoly profits is needed to induce sharing of early research than

[24] Note that when firms are not rivalrous (i.e., when $2\pi^D > \pi^M$), it is always in their joint interest to share and changes in patent policy do not have any impact on sharing.

[25] Patent policies that encourage lagging firms to exit may also be desirable when firms would otherwise have inadequate incentives to invest in the first place. Such environments provide the classic motivation for patent policy.

is needed for sharing of late stage research. An important insight of these results is that the prevalence of sharing in early stages of research in certain industries, often attributed to efficiencies of internalizing spillovers, could be due instead to competitive dynamics. In this case, the propensity to share in early stages would not indicate its higher social value.

Two important assumptions underlying these results are that lagging firms do not exit the race and that the firms are symmetric in their capabilities. When duopoly profits are too low to keep lagging firms in the race, the sharing dynamics are also shaped by investment decisions. We show that the firms may now be least likely to share early on because lagging firms are more likely to drop out of the race when they have made less progress. When firms are asymmetric in their research capabilities, this increasing sharing dynamic can again arise but for a different reason, a progress effect.

These results have implications for policy making in innovation environments. A first implication is that policy makers may want to distinguish between industries with different levels of duopoly profits and research costs since the sharing dynamics as well as the investment incentives are shaped by them. A second implication is that policy makers can encourage sharing by using broader patent protection and increasing the importance of the duplication effect relative to the rivalry effect. For example, in industries where sharing tends to break down as the firms approach the product market, broader protection of late stage sharing may be desirable. However, we show that broader patent protection of research outcomes may not always result in more sharing, and dynamic feedback effects must be taken into account. In particular, due to a progress effect, broader patent protection of late stage research can feed back to discourage sharing of early stage research. Our discussion of patent policies also reveals that if a social planner would like to induce sharing of both early and late stage research by using patent policies with the same breadth of protection for all stages of research, then the policy must be broader than what would be necessary to achieve sharing of early stage research only.

Our results suggest new directions for empirical research on innovation.[26] Although there is

[26] Recent work by Deck and Erkal (2013) using laboratory experiments finds confirmation of our monotonicity result in a two-stage game.

a large literature on research alliances, there has been little empirical research focusing on the dynamics of these alliances. Our theoretical work focuses on the dynamics of sharing where the intensity of product market competition, the difficulty of research, and the impatience of firms are the key factors. Future research could address whether these dynamics can be identified and empirically distinguished from the impact of other dynamic variables, such as the intensity of spillovers, financing issues, and the degree of antitrust risk, which are also likely shape the patterns of sharing. The role played by each factor may depend on the industry and the nature of the research.[27]

We conclude by mentioning two ways in which the current analysis can be extended. First, we allowed the firms to use fixed-fee contracts only. Suppose the firms can use contingent-fee licensing where the payment for sharing is structured as a share of profits earned in the product market. Such contracts may yield different sharing dynamics because the firms can use them to reduce the rivalry effect by inducing exit.[28] Hence, they can make it easier for the firms to achieve the constrained joint profit maximization benchmark of section 2.4.

Second, we assumed that the effort choices of the firms are discrete. In a continuous model, firms may have a stronger incentive not to share early on in the R&D race because it is easiest to get the lagging firm to reduce its research intensity early on. Thus, the sharing pattern (NS,S) might arise more often, even under Assumption 1. We have examined this using a numerical approach with various functional forms relating effort to the hazard rate and flow cost. The equilibria did not change significantly although we did find some examples where (NS,S) arose under Assumption 1. However, this occurred for values very close to the border of the parameter space represented by Assumption 1.

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[27] For example, Lerner and Merges (1998) find that in the biotechnology industry, it is the R&D firms' need for financing which may cause alliances to form at the earlier stages of research.

[28] For example, the firms can sign a profit-sharing contract at $(1,0)$ that induces the lagging firm to exit at $(2,1)$ and $(1,2)$. A more detailed discussion is available from the authors on request.

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Appendix

A The Value Functions

We present recursive equations for the value functions $V_1(h_1, h_2)$ and $U_1(h_1, h_2)$ for firm 1. The value functions for firm 2 are defined analogously.

Histories where no sharing decision is made. At (X, X) and (X, h) for $h = 0, 1, 2$, firm 1 is out of the game and its continuation profits are 0. That is, $V_1(X, X) = 0$ and $V_1(X, h) = 0$ for $h = 0, 1, 2$. At $(2, 2)$ and $(2, X)$, neither firm has any decisions. The continuation profits for firm 1 are $V_1(2, 2) = \int_0^\infty e^{-rt}\pi^D dt = \frac{\pi^D}{r} \equiv \widetilde{\pi}^D$ and $V_1(2, X) = \int_0^\infty e^{-rt}\pi^M dt = \frac{\pi^M}{r} \equiv \widetilde{\pi}^M$. At (h, X) for $h = 1$, 2, firm 1 has an investment decision and firm 2 is out of the game. The continuation profits for firm 1 are

$$V_1(h, X) = \max\left\{0, \int_0^\infty e^{-(\alpha+r)t}\left(\alpha V_1(h+1, X) - c\right) dt\right\}.$$

At (h, h) for $h = 0, 1$, the firms make investment decisions simultaneously. If firm 2 invests, firm 1's continuation profits are

$$V_1(h, h) = \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(h+1, h) + \alpha V_1(h, h+1) - c\right) dt\right\}.$$

If firm 2 does not invest, firm 1's continuation profits are $V_1(h, h) = \max\{0, V_1(h, X)\}$.

Histories where a sharing decision is made. At the asymmetric states (h_1, h_2), the value function $V_1(h_1, h_2)$ is the payoff before the sharing decision is made. If the sharing condition (1) holds, then $V_1(h_1, h_2)$ is defined as in (2). If the sharing condition does not hold, then $V_1(h_1, h_2) = U_1(h_1, h_2)$.

At $(h, 2)$ for $h = 0, 1$, after a decision not to share, only firm 1 has an investment decision. Its profits are

$$U_1(h, 2) = \max\left\{0, \int_0^\infty e^{-(\alpha+r)t}\left(\alpha V_1(h+1, 2) - c\right) dt\right\}.$$

At $(2, h)$ for $h = 0, 1$, after a decision not to share, only firm 2 has an investment decision. If firm 2 invests, the continuation profits for firm 1 are

$$U_1(2, h) = \int_0^\infty e^{-(\alpha+r)t}\left(\pi^M + \alpha V_1(2, h+1)\right) dt.$$

If firm 2 does not invest, firm 1's continuation profits are $U_1(2,h) = \widetilde{\pi}^M$.

At $(1,0)$ and $(0,1)$, after a decision not to share, both firms make simultaneous investment decisions. If firm 2 invests at $(1,0)$ or $(0,1)$, firm 1's continuation profits are

$$\begin{aligned} U_1(1,0) &= \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(2,0) + \alpha V_1(1,1) - c\right) dt\right\} \\ U_1(0,1) &= \max\left\{0, \int_0^\infty e^{-(2\alpha+r)t}\left(\alpha V_1(1,1) + \alpha V_1(0,2) - c\right) dt\right\}, \end{aligned}$$

respectively. If firm 2 does not invest at $(1,0)$ or $(0,1)$, firm 1's continuation profits are $U_1(1,0) = \max\{0, V_1(1,X)\}$ and $U_1(0,1) = \max\{0, V_1(0,X)\}$, respectively.

B Proof of Proposition 1

To solve for the MPE, we work backwards through the sharing conditions at the six asymmetric histories. We derive the equilibrium sharing conditions for $(1,0)$, $(2,0)$ and $(2,1)$. The three mirror histories $(0,1), (0,2)$, and $(1,2)$ have the same analysis.

The last sharing condition arises at $(2,1)$. The sharing condition (1) is

$$V_J(2,2) > U_J(2,1). \tag{10}$$

At $(2,2)$, each firm produces output in the product market. Their profits are

$$V_1(2,2) = V_2(2,2) = \widetilde{\pi}^D, \tag{11}$$

so joint profits are $V_J(2,2) = 2\widetilde{\pi}^D$. Joint profits under no sharing are

$$\begin{aligned} U_J(2,1) &= U_1(2,1) + U_2(2,1) \\ &= \frac{\pi^M + \alpha\widetilde{\pi}^D}{\alpha + r} + \frac{\alpha\widetilde{\pi}^D - c}{\alpha + r} = \frac{\pi^M + 2\alpha\widetilde{\pi}^D - c}{\alpha + r}. \end{aligned} \tag{12}$$

The sharing condition (10) simplifies to

$$\pi^M - c < 2\pi^D. \tag{13}$$

In part (iii), the sharing condition (13) holds, so the firms share step 2 at $(2,1)$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share step 1 at

$(2,0)$. The sharing condition (1) is $V_J(2,1) > U_J(2,0)$. Joint profits under sharing are $V_J(2,1) = V_J(2,2) = 2\widetilde{\pi}^D$ since the firms share at $(2,1)$ after sharing at $(2,0)$. Joint profits under no sharing are

$$\begin{aligned} U_J(2,0) &= U_1(2,0) + U_2(2,0) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c}{\alpha + r} = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r} = \frac{\pi^M + \alpha 2\widetilde{\pi}^D - c}{\alpha + r}. \end{aligned}$$

The sharing condition simplifies to $c > \pi^M - 2\pi^D$. This is condition (13), which holds, so the firms share step 1 at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > U_J(1,0)$. Joint profits under sharing are $V_J(1,1) = 2V_1(1,1)$, where

$$V_1(1,1) = \frac{\alpha V_1(1,2) + \alpha V_1(2,1) - c}{2\alpha + r} = \frac{\alpha V_J(2,1) - c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D - c}{2\alpha + r}. \tag{14}$$

Joint profits under no sharing are

$$U_J(1,0) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r} = \frac{2\alpha\widetilde{\pi}^D + \alpha V_J(1,1) - 2c}{2\alpha + r}.$$

Substituting for $U_J(1,0)$, the sharing condition (1) simplifies to

$$(2\alpha + r) V_J(1,1) > 2\alpha\widetilde{\pi}^D + \alpha V_J(1,1) - 2c. \tag{15}$$

Substituting for $V_J(1,1)$ using (14), the sharing condition simplifies to

$$\pi^D + c > 0, \tag{16}$$

which holds. Thus, in part (iii) there is always a unique MPE such that the firms share at $(2,1), (2,0)$ and $(1,0)$.

In parts (ii) and (iii), the sharing condition (13) fails, so the firms do not share at $(2,1)$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition (1) is $V_J(2,1) > U_J(2,0)$. Joint profits under no sharing are

$$\begin{aligned} U_J(2,0) &= U_1(2,0) + U_2(2,0) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c}{\alpha + r} = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}. \end{aligned}$$

The sharing condition simplifies to

$$V_J(2,1) > \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

Since the firms do not share at $(2,1)$, $V_J(2,1) = U_J(2,1)$ and we can substitute for $U_J(2,1)$ from (12). Simplifying gives $c > \pi^M - 2\pi^D$. This is the same as condition (13) which does not hold. Hence, the firms do not share step 1 at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > U_J(1,0)$. Joint profits under sharing are

$$\begin{aligned} V_J(1,1) &= 2V_2(1,1) \\ &= 2\frac{\alpha V_2(1,2) + \alpha V_2(2,1) - c}{2\alpha + r} = 2\frac{\alpha V_J(2,1) - c}{2\alpha + r} = 2\frac{\alpha(\pi^M + 2\alpha\widetilde{\pi}^D) - c(2\alpha + r)}{(2\alpha + r)(\alpha + r)}, \end{aligned}$$

where the last equality follows from (12) since the firms do not share at $(2,1)$ and $V_J(2,1) = U_J(2,1)$. Joint profits under no sharing are

$$U_J(1,0) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r}, \tag{17}$$

where, since there is no sharing at $(2,0)$,

$$V_J(2,0) = U_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r} = \frac{(2\alpha + r)\,\pi^M + 2\alpha^2\widetilde{\pi}^D - c\,(2\alpha + r)}{(\alpha + r)^2}.$$

The sharing condition simplifies to

$$c > (\pi^M - 2\pi^D) - \frac{2(\alpha + r)^2}{(2\alpha + r)^2}(\pi^M - \pi^D). \tag{18}$$

Since $\pi^M > \pi^D$, this condition is easier to satisfy than (13). Solving (18) for π^D gives

$$\frac{(2\alpha^2 - r^2)\pi^M - (2\alpha + r)^2 c}{2(3\alpha^2 + 2\alpha r)} < \pi^D.$$

This condition defines the boundary between parts (i) and (ii) in the proposition. For parameter values in part (ii), the sharing condition (18) holds, and there is a unique MPE such that the firms share at $(1,0)$ but not at $(2,1)$ or $(2,0)$. For parameter values in part (i), the sharing condition (18) fails and there is a unique MPE such that the firms do not share at $(1,0)$, $(2,0)$ or $(2,1)$.

C Calculation of the licensing fees

We consider parameter values satisfying part (iii) of Proposition 1. For these parameters, there is a unique MPE such that the firms share at every sharing history. The leading firm sets the licensing fee according to (3) so that the lagging firm is just indifferent between sharing and not sharing. At $(2,1)$, the licensing fee is

$$F(2,1) = V_2(2,2) - U_2(2,1) = \frac{\pi^D + c}{\alpha + r}, \tag{19}$$

where the last equality makes use of (11) and (12). At $(1,0)$, the licensing fee is

$$F(1,0) = V_2(1,1) - U_2(1,0).$$

We can substitute for $V_2(1,1)$ from (14). $U_2(1,0)$ is given by

$$U_2(1,0) = \frac{\alpha V_2(1,1) + \alpha V_2(2,0) - c}{2\alpha + r}. \tag{20}$$

We can again substitute for $V_2(1,1)$ from (14). Since the lagging firm has no bargaining power, its profit at $(2,0)$ is $V_2(2,0) = U_2(2,0)$ even though the firms share at $(2,0)$. Similarly, its profit at $(2,1)$ is $V_2(2,1) = U_2(2,1)$ even though the firms share at $(2,1)$. We have

$$V_2(2,0) = U_2(2,0) = \frac{\alpha V_2(2,1) - c}{\alpha + r} = \frac{\alpha U_2(2,1) - c}{\alpha + r} = \frac{\alpha^2 \widetilde{\pi}^D - c(2\alpha + r)}{(\alpha + r)^2},$$

where the last equality uses (12). Substituting for $V_2(2,0)$ in (20), $F(1,0)$ simplifies to

$$F(1,0) = \left(\frac{\pi^D + c}{\alpha + r}\right)\left(\frac{5 + 6\frac{r}{\alpha} + 2(\frac{r}{\alpha})^2}{4 + 8\frac{r}{\alpha} + 5(\frac{r}{\alpha})^2 + (\frac{r}{\alpha})^3}\right).$$

Comparing the fees $F(1,0)$ and $F(2,1)$, we find that $F(2,1) > F(1,0)$ if and only if $\frac{r}{\alpha}$ is above a cut-off of approximately $\frac{r}{\alpha} \cong 0.325$.

D Proof of Proposition 2

To save space, we do not present a complete proof.[29] Instead, we focus on part (ii) of the proposition where firm 1 is the leader. We show why the sharing pattern (NS,S) arises for some parameter values.

[29] The full proof of Proposition 2 is a generalization of Proposition 1 and is available on request.

We solve the game by working backwards through the sharing conditions. At $(2,1)$, the sharing condition (1) is $V_J(2,2) > U_J(2,1)$. Joint profits under no sharing are

$$U_J(2,1) = \frac{\pi^M + \alpha 2\widetilde{\pi}^D - c_2^2}{\alpha + r}.$$

Joint profits under sharing are $V_J(2,2) = 2\widetilde{\pi}^D$. The sharing condition simplifies to

$$2\pi^D - (\pi^M - c_2^2) > 0. \tag{21}$$

Similarly, the firms share at $(1,2)$ if and only if $2\pi^D - (\pi^M - c_1) > 0$. From now on, we assume both conditions hold, so the firms share at $(2,1)$ and $(1,2)$, and $V_J(2,1) = V_J(1,2) = 2\widetilde{\pi}^D$.

At $(2,0)$, the sharing condition (1) is $V_J(2,1) > U_J(2,0)$. Joint profits under no sharing are

$$U_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c_2^1}{\alpha + r}.$$

Joint profits under sharing are $V_J(2,1) = 2\widetilde{\pi}^D$. The sharing condition simplifies to $2\pi^D - (\pi^M - c_2^1) > 0$. From now on, we assume this holds so that the firms share at $(2,0)$. Similarly, the firms share at $(0,2)$ if and only if $2\pi^D - (\pi^M - c_1) > 0$. This is the same condition as the condition for sharing at $(1,2)$, so it holds. Hence, the firms share at $(2,0)$ and $(0,2)$. We have $V_J(2,0) = V_J(0,2) = 2\widetilde{\pi}^D$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > U_J(1,0)$. Joint profits under sharing are

$$V_J(1,1) = \frac{\alpha V_J(2,1) + \alpha V_J(1,2) - c_2^2 - c_1}{2\alpha + r} = \frac{4\alpha\widetilde{\pi}^D - c_2^2 - c_1}{2\alpha + r}.$$

Joint profits under no sharing are

$$U_J(1,0) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - c_2^1 - c_1}{2\alpha + r}.$$

Substituting for $V_J(2,0) = 2\widetilde{\pi}^D$ and $V_J(1,1)$, the sharing condition simplifies to

$$(2\pi^D + c_1 + c_2^2) + (c_2^1 - c_2^2)(\frac{2\alpha + r}{\alpha}) > 0. \tag{22}$$

When $c_2^1 - c_2^2 > 0$, the sharing condition (22) at $(1,0)$ holds trivially, so it is easier to satisfy than (21). However, there are parameters in this subregion such that $c_2^1 - c_2^2 < 0$ and the sharing condition fails. The firms do not share at $(1,0)$, so the sharing pattern is (NS,S).

E Proof of Proposition 3

The proposition considers parameters π^M, π^D, r, α, and c such that when $c_p^1 = c$, the firms do not share at any history. From the proof of Proposition (1), this is the case when condition (18) does not hold. That is:

$$c < (\pi^M - 2\pi^D) - \frac{2(\alpha+r)^2}{(2\alpha+r)^2}(\pi^M - \pi^D). \tag{23}$$

Condition (23) implies that (13) does not hold.

To solve for the MPE, we work backwards through the sharing conditions at the six asymmetric histories. We derive the equilibrium sharing conditions for $(1,0)$, $(2,0)$ and $(2,1)$. The three mirror histories $(0,1), (0,2)$, and $(1,2)$ have the same analysis.

At $(2,1)$, the sharing condition (1) is $V_J(2,2) > U_J(2,1)$. As in the proof of Proposition 1, this condition simplifies to (13). The parameter for early stage patent protection, c_p^1, does not impact this condition. Since (13) fails, the firms do not share at $(2,1)$.

Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition (1) is $V_J(2,1) > U_J(2,0)$. Joint profits under no sharing are

$$\begin{aligned} U_J(2,0) &= U_1(2,0) + U_2(2,0) \\ &= \frac{\pi^M + \alpha V_1(2,1)}{\alpha + r} + \frac{\alpha V_2(2,1) - c_p^1}{\alpha + r} = \frac{\pi^M + \alpha V_J(2,1) - c_p^1}{\alpha + r}. \end{aligned}$$

Substituting for $U_J(2,0)$, the sharing condition simplifies to $c_p^1 > \pi^M - rV_J(2,1)$. Since the firms do not share at $(2,1)$, we have $V_J(2,1) = U_J(2,1)$ and we can substitute for $U_J(2,1)$ from (12). Simplifying, we get

$$c_p^1 > \frac{\alpha}{\alpha + r}(\pi^M - 2\pi^D) + \frac{r}{\alpha + r}c. \tag{24}$$

For $c_p^1 = c$, this condition simplifies to the sharing condition (13) which does not hold. For c_p^1 sufficiently large, (24) holds. We define the threshold level $\bar{\bar{c}}_p^1$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(2,0)$.

Parts (i) and (ii). We assume that the patent policy c_p^1 is too weak to induce sharing at $(2,0)$. That is, $c_p^1 < \bar{\bar{c}}_p^1$. At $(1,0)$, the sharing condition (1) is $V_J(1,1) > U_J(1,0)$. Joint profits under no sharing are

$$U_J(1,0) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - c - c_p^1}{2\alpha + r}.$$

The sharing condition simplifies to $(\alpha + r) V_J(1,1) > \alpha V_J(2,0) - c - c_p^1$. Since $V_J(1,1) = 2V_2(1,1)$, we have

$$V_J(1,1) = 2\frac{\alpha V_2(1,2) + \alpha V_2(2,1) - c}{2\alpha + r} = 2\frac{\alpha V_J(2,1) - c}{2\alpha + r}. \tag{25}$$

Since the firms do not share at $(2,0)$, we have

$$V_J(2,0) = U_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c_p^1}{\alpha + r}.$$

Substituting for $V_J(1,1)$ and $V_J(2,0)$, the sharing condition at $(1,0)$ simplifies to

$$\alpha r(3\alpha + 2r) V_J(2,1) > r(\alpha + r)c + \alpha(2\alpha + r)\pi^M - (2\alpha + r)^2 c_p^1.$$

Since the firms do not share at $(2,1)$, we can substitute for $V_J(2,1) = U_J(2,1)$ from (12). Then, the sharing condition at $(1,0)$ simplifies to

$$c_p^1 > \frac{\alpha}{(\alpha + r)} \frac{(2\alpha^2 - r^2)}{(2\alpha + r)^2} (\pi^M - 2\pi^D) - \frac{\alpha(\alpha + r)}{(2\alpha + r)^2} 2\pi^D + \frac{r}{(\alpha + r)} c.$$

We can rewrite this as

$$c_p^1 > \frac{\alpha}{(\alpha + r)} (\pi^M - 2\pi^D) - \frac{2\alpha(\alpha + r)}{(2\alpha + r)^2} (\pi^M - \pi^D) + \frac{r}{(\alpha + r)} c. \tag{26}$$

For $c_p^1 = c$, condition (26) simplifies to condition (18) which does not hold. However, as the patent policy is strengthened to $c_p^1 > c$, (26) may start to hold. We define $\bar{c}_p^1 > c$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(1,0)$. Comparing (26) with (24), it is clear that $\bar{c}_p^1 < \bar{\bar{c}}_p^1$. Therefore, as the policy maker increases the strength of early patent protection, the firms start to share at $(1,0)$ before they start to share at $(2,0)$.

We have shown that for a patent policy c_p^1 with $\bar{c}_p^1 > c_p^1 > c$ as in part (ii), there is a unique MPE such that the firms do not share at any history. For a patent policy c_p^1 with $\bar{\bar{c}}_p^1 > c_p^1 > \bar{c}_p^1$

as in part (i), there is a unique MPE such that the firms share at $(1,0)$, but they do not share at either $(2,0)$ or $(2,1)$.

Part (iii). We assume that the patent policy c_p^1 is strong enough to induce sharing at $(2,0)$. That is, $c_p^1 > \overline{\overline{c}}_p^1$. The firms share at $(2,0)$, but they do not share at $(2,1)$. At $(1,0)$, the sharing condition is $V_J(1,1) > U_J(1,0)$. The joint profits under no sharing are

$$U_J(1,0) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - c - c_p^1}{2\alpha + r} = \frac{\alpha V_J(2,1) + \alpha V_J(1,1) - c - c_p^1}{2\alpha + r}$$

where the last equality uses the fact that the firms share at $(2,0)$. Substituting for $U_J(1,0)$, the sharing condition simplifies to $(\alpha + r) V_J(1,1) > \alpha V_J(2,1) - c - c_p^1$. Substituting for $V_J(1,1)$ from (25), the sharing condition simplifies to $\alpha r V_J(2,1) > rc - (2\alpha + r)c_p^1$. Since the firms do not share $(2,1)$, we can substitute for $V_J(2,1) = U_J(2,1)$ using (12). The sharing condition at $(1,0)$ then simplifies to

$$c_p^1 > \frac{r}{(\alpha + r)} c - \frac{\alpha r}{(\alpha + r)(2\alpha + r)} \pi^M - \frac{\alpha^2}{(\alpha + r)(2\alpha + r)} 2\pi^D. \qquad (27)$$

This holds because $c_p^1 > c$. We have shown that when $c_p^1 > \overline{\overline{c}}_p^1$, there is a unique MPE such that the firms share at $(1,0)$ and $(2,0)$, but they do not share at $(2,1)$.

F Proof of Proposition 4

The proposition considers parameters π^M, π^D, r, α, and c such that when $c_p^2 = c$, the firms share at $(1,0)$, but they do not share at $(2,0)$ and $(2,1)$. The proposition also assume that $c < \overline{c}$ where $\overline{c} = (\pi^M - 2\pi^D) - \frac{2(\alpha+r)}{(4\alpha+3r)}(\pi^M - \pi^D)$. This subregion of parameters is described by the following condition:

$$(\pi^M - 2\pi^D) - \frac{2(\alpha + r)^2}{(2\alpha + r)^2}(\pi^M - \pi^D) < c < (\pi^M - 2\pi^D) - \frac{2(\alpha + r)}{(4\alpha + 3r)}(\pi^M - \pi^D) = \overline{c}. \qquad (28)$$

The first inequality in (28) is condition (18) from the proof of Proposition 1. When $c_p^1 = c$, this inequality implies that the firms share at $(1,0)$ but not at $(2,1)$ or $(2,0)$. The second inequality in (28) is an assumption that will be used below.

To solve for the MPE, we work backwards through the sharing conditions at the six asymmetric histories. We derive the equilibrium sharing conditions for $(1,0)$, $(2,0)$ and $(2,1)$. The three mirror histories $(0,1), (0,2)$, and $(1,2)$ have the same analysis.

At $(2,1)$, the sharing condition is $V_J(2,2) > U_J(2,1)$. Joint profits under sharing are $V_J(2,2) = 2\widetilde{\pi}^D$. Joint profits under no sharing are

$$U_J(2,1) = U_1(2,1) + U_2(2,1) = \frac{\pi^M + 2\alpha\widetilde{\pi}^D - c_p^2}{\alpha + r}. \tag{29}$$

Substituting for $V_J(2,2)$ and $U_J(2,1)$, the sharing condition simplifies to

$$c_p^2 > \pi^M - 2\pi^D. \tag{30}$$

We define the threshold level $\bar{\bar{c}}_p^2 = \pi^M - 2\pi^D$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(2,1)$.

Parts (i) and (ii). We first consider patent policies that are not strong enough to induce sharing at $(2,1)$. That is, $c_p^2 < \bar{\bar{c}}_p^2 = \pi^M - 2\pi^D$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share at $(2,0)$. The sharing condition at $(2,0)$ is $V_J(2,1) > U_J(2,0)$. Joint profits are under no sharing are

$$U_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

Substituting for $U_J(2,0)$, the sharing condition at $(2,0)$ simplifies to $rV_J(2,1) > \pi^M - c$. Since the firms do not share at $(2,1)$, we have $V_J(2,1) = U_J(2,1)$. Substituting for $U_J(2,1)$ from (29), the sharing condition at $(2,0)$ simplifies to

$$\left(1 + \frac{r}{\alpha}\right) c - c_p^2 \frac{r}{\alpha} > \pi^M - 2\pi^D.$$

This condition fails because $c > c_p^2$ and $c_p^2 < \pi^M - 2\pi^D$, so the firms do not share at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > U_J(1,0)$. Joint profits under no sharing are

$$U_J(1,0) = \frac{\alpha V_J(2,0) + \alpha V_J(1,1) - 2c}{2\alpha + r}.$$

The sharing condition simplifies to $(\alpha + r) V_J(1,1) > \alpha V_J(2,0) - 2c$. We can substitute for $V_J(1,1)$ using (25). Since the firms do not share at $(2,0)$, we have

$$V_J(2,0) = U_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

The sharing condition at $(1,0)$ simplifies to

$$\alpha r(3\alpha + 2r)V_J(2,1) > \alpha(2\alpha + r)\pi^M - \alpha(4\alpha + 3r)c. \tag{31}$$

Since the firms do not share at $(2,1)$, we can substitute for $V_J(2,1)$ from (29). The sharing condition at $(1,0)$ simplifies to

$$c_p^2 < c\frac{(\alpha + r)(4\alpha + 3r)}{r(3\alpha + 2r)} + 2\pi^D\frac{\alpha}{r} - \pi^M\frac{(2\alpha^2 - r^2)}{r(3\alpha + 2r)}. \tag{32}$$

We define the threshold level $\bar{c}_p^2$ to be the patent policy such that the firms are indifferent between sharing and not sharing at $(1,0)$. It is straightforward but tedious to show that when (28) holds, the threshold $\bar{c}_p^2$ satisfies $c < \bar{c}_p^2 < (\pi^M - 2\pi^D)$.

We have shown that for patent policies c_p^2 with $c < c_p^2 < \bar{c}_p^2$ as in part (ii), there is a unique MPE in which the firms share at $(1,0)$ but not at $(2,0)$ or $(2,1)$. For stronger patent policies c_p^2 with $\bar{c}_p^2 < c_p^2 < \bar{\bar{c}}_p^2$ as in part (i), the firms do not share at any history.

Part (iii). We next consider patent policies that are strong enough to induce sharing at $(2,1)$. That is, $c_p^2 > \bar{\bar{c}}_p^2 = \pi^M - 2\pi^D$. Before considering the sharing decision at $(1,0)$, we need to see whether the firms share step 1 at $(2,0)$. The sharing condition at $(2,0)$ is $V_J(2,1) > U_J(2,0)$. Joint profits are under no sharing are

$$U_J(2,0) = \frac{\pi^M + \alpha V_J(2,1) - c}{\alpha + r}.$$

Substituting for $U_J(2,0)$, the sharing condition simplifies to $rV_J(2,1) > \pi^M - c$. Since the firms share at $(2,1)$, we have $V_J(2,1) = 2\tilde{\pi}^D$. Substituting for $V_J(2,1)$, the sharing condition simplifies to $c > \pi^M - 2\pi^D$. From (28), this fails, so the firms do not share at $(2,0)$.

At $(1,0)$, the sharing condition (1) is $V_J(1,1) > U_J(1,0)$. Following the analysis in parts (i) and (ii), the sharing condition simplifies to (31). Substituting for $V_J(2,1) = 2\tilde{\pi}^D$, the sharing condition at $(1,0)$ simplifies to

$$c > (\pi^M - 2\pi^D) - \frac{2(\alpha + r)}{(4\alpha + 3r)}(\pi^M - \pi^D).$$

From (28), this fails so the firms do not share at $(1,0)$. When $c_p^2 > \bar{\bar{c}}_p^2$ as in part (iii), there is a unique MPE in which the firms share at $(2,1)$, but they do not share at $(2,0)$ or $(1,0)$.